Lineare Finite-Elemente Analyse einer Balkenstruktur

Probemstellung: Berechnung der Systemmatrizen

Berechnung aller Systematrizen der linearisierten Bewegungsgleichungen. Die Eingabedaten und die Element-Matrizen muessen vorliegen. Sie werden mit *1_Definition Balkenstruktur.nb* und *0_0BalkenEle 2D.nb* festgelegt.

```
FEName
```

```
Elast. Winkel mit 3 Elementen
```

Loesung: 2 Berechnung der Systemmatrizen fuer 2D-Strukturen

■ 2.1 FE-Dimension nF, Knotenvektor ZF==ZF0, Drehmatrizen Gammae, Sammelmatrizen Te

```
nfk   (*  Zahl der FHG pro Knoten *)
```

```
3
```

Zahl der FE-Systemkoordinaten

```
nF = nfk * nK
```

```
12
```

FE-Systemkoordinaten ZF, wenn Struktur unverformt = Knotenkkoordinaten bezgl. Referenzsystem

```
ZF = Table[0,{nF}];
Do [ZF[[nfk*e-2]] = RK[[e,1]];
    ZF[[nfk*e-1]] = RK[[e,2]],{e,1,nK}];
ZF
```

```
{0, 0, 0, 1, 0, 0, 2, 0, 0, 2, 0.4, 0}
```

■ Drehmatrizen Gammae aller Elemente

```
Gammae = Table[0,{nE},{3},{3}];
Do [ Ak  = Inde[[e,1]]; Bk = Inde[[e,2]];
     dr = RK[[Bk]] - RK[[Ak]];
     leh = Edata[[e,5]];
     cg = dr[[1]] / leh;
     sg = dr[[2]] / leh;
     Gammae[[e,1,1]] = cg;
     Gammae[[e,2,2]] = cg;
     Gammae[[e,1,2]] = sg;
     Gammae[[e,2,1]] =-sg;
     Gammae[[e,3,3]] = 1;                 ;
     Print["Gammae fuer e = ",e];
     Print[MatrixForm[Gammae[[e]]]];
     ,{e,nE}];
```

```
General::spell1 : Possible spelling error: new symbol name "Gammae" is similar to existing symbol "Gamma".
```

Gammae fuer e = 1

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Gammae fuer e = 2

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Gammae fuer e = 3

$$\begin{pmatrix} 0 & 1. & 0 \\ -1. & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

■ IndexTafel in zF je Element: Index-1 fuer Knoten A und B im Vektor zF

```
Indufe = Table[0,{nE},{2}];
Do [Ak  = Inde[[e,1]]; Bk = Inde[[e,2]];
    Indufe[[e,1]] = nfk*Ak - 3;
    Indufe[[e,2]] = nfk*Bk - 3;,{e,nE}];
MatrixForm[Indufe]
```

$$\begin{pmatrix} 0 & 3 \\ 3 & 6 \\ 6 & 9 \end{pmatrix}$$

■ Sammelmatrizen TeT (=Transponierte von Te) aller Elemente

```
TeT = Table[0,{nE},{nF},{6}];
Do [ Do [ Do [
      TeT [[e,i+Indufe[[e,1]],j  ]] = Gammae[[e,j,i]];
      TeT [[e,i+Indufe[[e,2]],j+3]] = Gammae[[e,j,i]];,{i,3}],{j,3}];
      Print["Te fuer e = ",e];
      If[keyPrint == 1, Print[MatrixForm[Transpose[TeT[[e]]]]] ];,{e,nE}];
```

Te fuer e = 1

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Te fuer e = 2

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

Te fuer e = 3

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1. & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1. & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1. & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1. & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

■ 2.2 Berechnung der sym. Steifigkeitsmatrix KF, sym. Massenmatrix MF und CFt = TeT . Cte . Gamae

```
KF = Table[0, {nF}, {nF}];
Do[KF = KF + TeT⟦e⟧.Ke.Transpose[TeT⟦e⟧] /.
     {le → Edata⟦e, 5⟧, Ae → Edata⟦e, 2⟧, I33e → Edata⟦e, 3⟧, Ee → Edata⟦e, 1⟧};
  , {e, nE}];
If[keyPrint == 1, MatrixForm[KF]]
```

$$\begin{pmatrix} 1.26\times10^{8} & 0 & 0 & -1.26\times10^{8} & 0 & 0 & 0 & 0 & 0 & 0 & \\ 0 & 10206. & 5103. & 0 & -10206. & 5103. & 0 & 0 & 0 & 0 & \\ 0 & 5103. & 3402. & 0 & -5103. & 1701. & 0 & 0 & 0 & 0 & \\ -1.26\times10^{8} & 0 & 0 & 2.52\times10^{8} & 0 & 0 & -1.26\times10^{8} & 0 & 0 & 0 & \\ 0 & -10206. & -5103. & 0 & 20412. & 0. & 0 & -10206. & 5103. & 0 & \\ 0 & 5103. & 1701. & 0 & 0. & 6804. & 0 & -5103. & 1701. & 0 & \\ 0 & 0 & 0 & -1.26\times10^{8} & 0 & 0 & 1.26563\times10^{8} & 0 & -112534. & -562669. & \\ 0 & 0 & 0 & 0 & -10206. & -5103. & 0 & 9.9751\times10^{8} & -5103. & 0 & -9. \\ 0 & 0 & 0 & 0 & 5103. & 1701. & -112534. & -5103. & 33411. & 112534. & \\ 0 & 0 & 0 & 0 & 0 & 0 & -562669. & 0 & 112534. & 562669. & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -9.975\times10^{8} & 0 & 0 & 9.9 \\ 0 & 0 & 0 & 0 & 0 & 0 & -112534. & 0 & 15004.5 & 112534. & \end{pmatrix}$$

```
MF = Table[0, {nF}, {nF}];
Do[MF = MF + TeT[[e]].Me.Transpose[TeT[[e]]] /.
     {me → Edata[[e, 6]], le → Edata[[e, 5]], A → Edata[[e, 2]], I33 → Edata[[e, 3]]};
  , {e, nE}];
If[keyPrint == 1, MatrixForm[MF]]
```

1.8	0	0	0.9	0	0	0	0	0	0	(
0	2.00571	0.282857	0	0.694286	-0.167143	0	0	0	0	(
0	0.282857	0.0514286	0	0.167143	-0.0385714	0	0	0	0	(
0.9	0	0	3.6	0	0	0.9	0	0	0	(
0	0.694286	0.167143	0	4.01143	0.	0	0.694286	-0.167143	0	(
0	-0.167143	-0.0385714	0	0.	0.102857	0	0.167143	-0.0385714	0	(
0	0	0	0.9	0	0	4.02865	0	-0.125718	0.771454	(
0	0	0	0	0.694286	0.167143	0	4.00578	-0.282857	0	1.0(
0	0	0	0	-0.167143	-0.0385714	-0.125718	-0.282857	0.0605717	-0.0742882	(
0	0	0	0	0	0	0.771454	0	-0.0742882	2.22865	(
0	0	0	0	0	0	0	1.00003	0	0	2.0(
0	0	0	0	0	0	0.0742882	0	-0.00685737	0.125718	(

```
CFt = Table[0, {nF}, {3}];
Do[
  CFt = Chop[CFt + TeT[[e]].Cte.Gammae[[e]] /. {le → Edata[[e, 5]], me → Edata[[e, 6]]}];
  , {e, 1, nE}];
MatrixForm[CFt]
```

2.7	0	0
0	2.7	0
0	0.45	0
5.4	0	0
0	5.4	0
0	0	0
5.7001	0	0
0	5.7001	0
-0.200007	-0.45	0
3.0001	0	0
0	3.0001	0
0.200007	0	0