

Lineare Finite-Elemente Analyse einer Balkenstruktur

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Problemstellung: Lineare Statik-Analyse

Definition von Lagerungen (Einspannung im Knoten 1) und Lasten (Erdbeschleunigung g in negativer 2-Richtung sowie Kraft F in 1 am Knoten 4).

Berechnung der reduzierten Bewegungsgleichungen, ohne geometrische Steifigkeiten $KF0$

Loesen der linearen Statik $KFy \cdot yF = hFy$ und Auswertung der Lagerkrafte, Normalspannungen und Biegespannungen fuer $F = 0, -200, 200 \text{ N}$.

Die Abschnitte 1 und 2 muessen bereits durchgerechnet sein!

Aufgaben:

3.1 Bestimmung der Minimalkoordinaten yF

Bilden des Lastvektors hF infolge Knotenlasten

Bilden von hF infolge der Referenzbewegung

Reduzierung auf Minimalform der lin. DGL: $MFy \cdot ddyF + DFy \cdot dyF + KFy \cdot yF = hFy$

3.2 Berechne die Knotenverschiebungen $zFstat$

Berechne die Auflagerkraft $fbar$

Berechne die Schnittgroessen $f1e$ und $m3e$ je Element

Berechne die Normalspannung und Biegespannung je Element

Stelle die Ergebnisse grafisch dar.

FEName

Elast. Winkel mit 3 Elementen

Festlegung der Parameter

- Setzen von Lagerbedingungen: **Lager** = $\{\{\text{Nr. Knoten mit Lager}, \text{Key-u1}, \text{Key-u2}, \text{Key-theta3}\}\}$,
Key: 0=frei, 1=fest der drei Bewegungen $u1, u2, \text{theta3}$

Lager = $\{\{1, 1, 1, 1\}\}$

$\{\{1, 1, 1, 1\}\}$

- Setzen von Knotenlasten: **LastKnot** = $\{\{\text{Nr. Knoten mit Last}, F1, F2, M3\}\}$,
Werte fuer Kraefte $F1, F2$, Moment $M3$

LastKnot = $\{\{4, F, 0, 0\}\}$

$\{\{4, F, 0, 0\}\}$

- Setzen einer Referenzbewegung: `LastRef = {a1, a2, domega3, omega3}`,
Werte fuer Linearbeschleunigung a1, a2, Winkelbeschleunigung domega3, Winkelgeschwindigkeit omega3
Beachte: Fuer Erdbeschleunigung gilt $a == -g$!
domega3 und omega3 sind nicht implementiert!

```
LastRef = {0, 9.81, 0, 0}
```

```
{0, 9.81, 0, 0}
```

- Setzen von Berechnungsschlusseln

```
keyPrint = 1; (* 0 = kein Print von System-Matrizen, 1 = Print *)
```

Loesung: 3 Lineare Statik Analyse

- 3.1 Bestimmung der Minimalkoordinaten y_F und der Minimal-Systemmatrizen

```
nKLager = Length[Lager];
nIndr = Sum[Delete[Transpose[Lager], 1][[j, i]], {i, nKLager}, {j, nfk}];
Indr = Table[0, {nIndr}];
nR=0;
If[Lager[[1,1]]==0, IndM = Table[n, {n, 1, nF}]; nM = nF;, (* keine Randbedingungen *)
(* mit Randbedingungen *)
Do[If[Lager[[k,2]]==1, nR=nR+1; Indr[[nR]]=(Lager[[k,1]]-1)*nfk+1];
If[Lager[[k,3]]==1, nR=nR+1; Indr[[nR]]=(Lager[[k,1]]-1)*nfk+2];
If[Lager[[k,4]]==1, nR=nR+1; Indr[[nR]]=(Lager[[k,1]]-1)*nfk+3];
, {k, 1, nKLager}];];
nR
Indr
(* Zahl der gebundenen Koorditen *)
(* gebundene Koordinaten *)

3

{1, 2, 3}

If[nR==0, IndM = Table[n, {n, 1, nF}],
IndM = Delete[Table[n, {n, 1, nF}], Partition[Indr, 1]]];
nM = nF - nR (* Zahl der (freien) Minimal-Koordinaten *)
IndM (* Liste der Minimal-Koordinaten *)

9

{4, 5, 6, 7, 8, 9, 10, 11, 12}
```

- Bilden des Lastvektors h_F infolge Knotenlasten

```
nKLast = Length[LastKnot];
hF = Table[0, {nF}];
Do[Do[ hF[[LastKnot[[kk,1]]*nfk-nfk+k]] = hF[[LastKnot[[kk,1]]*nfk-nfk+k]] + LastKnot[[kk,1+k]]
, {k, nfk}];
, {kk, nKLast}];
hF

{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, F, 0, 0}
```

- Bilden von h_F infolge der Referenzbewegung

a-Anteil in $h_{Fref} = -C_F t \cdot a$

```
Beschl = Delete[ReplacePart[LastRef, 0, 3], 4]

{0, 9.81, 0}
```

```
hFref = - Cft . Besch1
```

```
{0., -26.487, -4.4145, 0., -52.974, 0., 0., -55.918, 4.4145, 0., -29.431, 0.}
```

■ Reduzierung auf Minimalform der lin. DGL.: $\text{MFy} \cdot \text{ddyF} + \text{DFy} \cdot \text{dyF} + \text{KFy} \cdot \text{yF} = \text{hFy}$

```
hFsum = Chop[hF + hFref]
```

```
{0, -26.487, -4.4145, 0, -52.974, 0, 0, -55.918, 4.4145, F, -29.431, 0}
```

```
hFy = Table[0, {nM}];
```

```
Do [ hFy[[i]] = hFsum[[IndM[[i]]]], {i, 1, nM}]; hFy
```

```
{0, -52.974, 0, 0, -55.918, 4.4145, F, -29.431, 0}
```

```
KFy = Table[0, {nM}, {nM}];
```

```
Do [ Do [KFy[[i,j]] = KF[[IndM[[i]], IndM[[j]]]], {j, 1, nM}], {i, 1, nM}];
```

```
If[keyPrint == 1, KFy//MatrixForm]
```

$$\begin{pmatrix} 2.52 \times 10^8 & 0 & 0 & -1.26 \times 10^8 & 0 & 0 & 0 & 0 & 0 \\ 0 & 20412. & 0. & 0 & -10206. & 5103. & 0 & 0 & 0 \\ 0 & 0. & 6804. & 0 & -5103. & 1701. & 0 & 0 & 0 \\ -1.26 \times 10^8 & 0 & 0 & 1.26563 \times 10^8 & 0 & -112534. & -562669. & 0 & -112534. \\ 0 & -10206. & -5103. & 0 & 9.9751 \times 10^8 & -5103. & 0 & -9.975 \times 10^8 & 0 \\ 0 & 5103. & 1701. & -112534. & -5103. & 33411. & 112534. & 0 & 15004.5 \\ 0 & 0 & 0 & -562669. & 0 & 112534. & 562669. & 0 & 112534. \\ 0 & 0 & 0 & 0 & -9.975 \times 10^8 & 0 & 0 & 9.975 \times 10^8 & 0 \\ 0 & 0 & 0 & -112534. & 0 & 15004.5 & 112534. & 0 & 30009. \end{pmatrix}$$

```
MFy = Table[0, {nM}, {nM}];
```

```
Do [ Do [MFy[[i,j]] = MF[[IndM[[i]], IndM[[j]]]], {j, 1, nM}], {i, 1, nM}];
```

```
If[keyPrint == 1, MFy//MatrixForm]
```

$$\begin{pmatrix} 3.6 & 0 & 0 & 0.9 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4.01143 & 0. & 0 & 0.694286 & -0.167143 & 0 & 0 & 0 \\ 0 & 0. & 0.102857 & 0 & 0.167143 & -0.0385714 & 0 & 0 & 0 \\ 0.9 & 0 & 0 & 4.02865 & 0 & -0.125718 & 0.771454 & 0 & 0.0742882 \\ 0 & 0.694286 & 0.167143 & 0 & 4.00578 & -0.282857 & 0 & 1.00003 & 0 \\ 0 & -0.167143 & -0.0385714 & -0.125718 & -0.282857 & 0.0605717 & -0.0742882 & 0 & -0.00685737 \\ 0 & 0 & 0 & 0.771454 & 0 & -0.0742882 & 2.22865 & 0 & 0.125718 \\ 0 & 0 & 0 & 0 & 1.00003 & 0 & 0 & 2.00007 & 0 \\ 0 & 0 & 0 & 0.0742882 & 0 & -0.00685737 & 0.125718 & 0 & 0.00914316 \end{pmatrix}$$

■ 3.2 Berechnung der Statikloesung $F = 0$

```
F1 = 0
```

```
0
```

■ Knotenverschiebungen $\text{zFstat} = \text{Jy} \cdot \text{yF}$, $\text{yF} = \text{KFy}^{-1} \cdot \text{hFy}$,

```
nfk
```

```
3
```

```
yF = Inverse[KFy] . hFy/.F->F1;
```

```
zFstat = Chop[yF];
```

```
If[nR>0, Do [zFstat = Insert[zFstat,0,Indr[[j]] ], {j,nR}];
```

```
zFstat
```

```
{0, 0, 0, 0, -0.101793, -0.17648, 0, -0.309128, -0.221465, 0.088586, -0.309128, -0.221465}
```

```
Print["Knoten k | u1 (m) | u2 (m) | theta3 (rad) "];
Do[Print[k, " | ", zFstat[[ (k - 1) * nfk + 1]],
" | ", zFstat[[ (k - 1) * nfk + 2]], " | ", zFstat[[ (k - 1) * nfk + 3]]];
,
{k,
nK}];
```

Knoten k	u1 (m)	u2 (m)	theta3 (rad)
1	0	0	0
2	0	-0.101793	-0.17648
3	0	-0.309128	-0.221465
4	0.088586	-0.309128	-0.221465

■ Auflagerkraefte in der Bedeutung der Koordinaten nach Indr: $\mathbf{fbar} = \mathbf{JFbar}^T (\mathbf{KF} \cdot \mathbf{zFstat} - \mathbf{hF})$

```
fbar = KF . zFstat - hFsum;
fbar = Chop[Expand[Delete[fbar, Partition[IndM, 1]]]]; MatrixForm[fbar]
```

$$\begin{pmatrix} 0 \\ 164.81 \\ 223.672 \end{pmatrix}$$

■ Schnittgröoessen je Element: $\mathbf{sigmae} = \mathbf{He} \cdot \mathbf{epse} = \mathbf{He} \cdot \mathbf{Be} \cdot \mathbf{Te} \cdot \mathbf{zF}$

```
sigmae = Table[0, {nE}, {2}];
Print["Element e | fle (N) | m3e (Nm) | me3(0) | me3(1) "];
Do[
sigmae[[e]] = Chop[Expand[He.Be.Transpose[TeT[[e]]].zFstat /.
{le → Edata[[e, 5]], Ae → Edata[[e, 2]], I33e → Edata[[e, 3]], Ee → Edata[[e, 1]]}]];
Print[e, " | ", sigmae[[e, 1]], " | ", sigmae[[e, 2]], " | ",
sigmae[[e, 2]] /. xi → 0, " | ", Chop[sigmae[[e, 2]] /. xi → 1]];
,
{e,
nE}]
```

Element e	fle (N)	m3e (Nm)	me3(0)	me3(1)
1	0	-219.257 + 138.323 xi	-219.257	-80.9345
2	0	-80.9345 + 85.349 xi	-80.9345	4.4145
3	-29.431	0	0	0

■ Normalspannung = $\mathbf{sigmae}[[1]]/\mathbf{Ae}$ und Biegespannung = $\mathbf{sigmae}[[2]]/\mathbf{I33e} \cdot \mathbf{he}$

```
spannung = Table[0, {nE}, {2}];
Print["Element e | Normalspg (N/m^2) | Biegespg (N/m^2) "];
Do[
spannung[[e, 1]] =
Chop[sigmae[[e, 1]]] / Ae /. {le → Edata[[e, 5]], Ae → Edata[[e, 2]], I33e → Edata[[e, 3]], Ee → Edata[[e, 1]]};
spannung[[e, 2]] = Expand[sigmae[[e, 2]]] / I33e * h1 /.
{le → Edata[[e, 5]], Ae → Edata[[e, 2]], I33e → Edata[[e, 3]], h1 → Edata[[e, 7]]};
Print[e, " | ", spannung[[e, 1]], " | ", spannung[[e, 2]]];
, {e, nE}]
```

Element e	Normalspg (N/m^2)	Biegespg (N/m^2)
1	0	$-1.62413 \times 10^8 + 1.02461 \times 10^8 \text{ xi}$
2	0	$-5.99515 \times 10^7 + 6.32215 \times 10^7 \text{ xi}$
3	-15490.	0

■ Grafiken für Verschiebung und Biegemoment

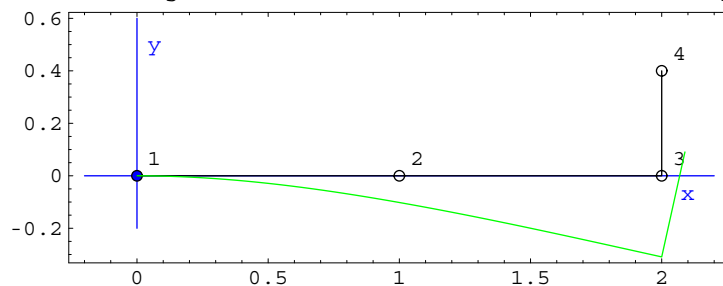
```

m3eSka = 1000;
fleSka = 500;
Ne3 = Join[Ne, {Table[0, {nFe}]}]; (* Nullzeile anhaengen *)
Ue = Table[0, {nE}, {3}];
plotUe = Table[0, {nE}];
plotf1 = Table[0, {nE}];
plotMe = Table[0, {nE}];
Do[Ue[[e]] = Transpose[Gammmae[[e]].Ne3.Transpose[TeT[[e]]].(zFstat + ZF) /. {le → Edata[[e, 5], F → F1}];
  plotUe[[e]] = ParametricPlot[{Ue[[e, 1]], Ue[[e, 2]]}, {xi, 0, 1},
    DisplayFunction → Identity, PlotStyle → {{RGBColor[0, 1, 0]}}];

Res = Transpose[Gammmae[[e]].{xi le,  $\frac{\text{sigmae}[[e, 2]]}{m3eSka}$ , 0} + RK[[e]] /. {le → Edata[[e, 5], F → F1}];
plotMe[[e]] = ParametricPlot[{Res[[1]], Res[[2]]}, {xi, 0, 1},
  DisplayFunction → Identity, PlotStyle → {{RGBColor[1, 0, 0]}}];
Res = Transpose[Gammmae[[e]].{xi le, sigmae[[e, 1]] / fleSka, 0} + RK[[e]] /. {le → Edata[[e, 5], F → F1}];
plotf1[[e]] = ParametricPlot[{Res[[1]], Res[[2]]}, {xi, 0, 1},
  DisplayFunction → Identity, PlotStyle → {{RGBColor[1, 0, 0]}}];
, {e, nE}];
Show[PlStructure1, PlStructure2, PlStructure3, PlStructure4, plotUe, DisplayFunction → $DisplayFunction,
  PlotRange → All, PlotLabel → {"Knotenverschiebungen:", FEName, "Last g & F = ", F1}];

```

otenverschiebungen:, Elast. Winkel mit 3 Elementen, Last g & F = ,

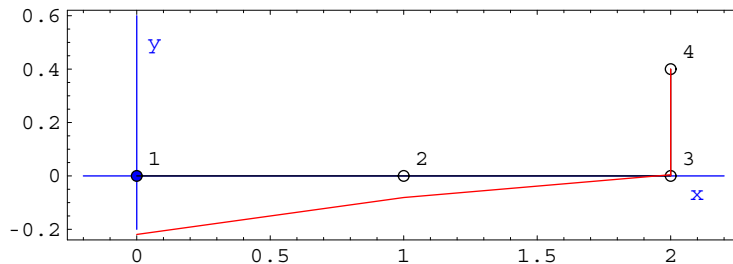


```

Show[PlStructure1, PlStructure2, PlStructure3, PlStructure4, plotMe,
  DisplayFunction → $DisplayFunction, PlotRange → All,
  PlotLabel → {"Biegemoment *", m3eSka, "Nm:", FEName, "Last g & F = ", F1}];

```

gemoment *, 1000, Nm:, Elast. Winkel mit 3 Elementen, Last g & F =

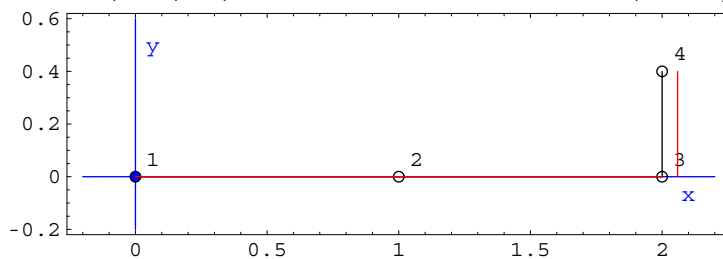


```

Show[PlStructure1, PlStructure2, PlStructure3, PlStructure4, plotf1,
  DisplayFunction → $DisplayFunction, PlotRange → All,
  PlotLabel → {"Normalkraft *", fleSka, "N:", FEName, "Last g & F = ", F1}];

```

rmalkraft *, 500, N:, Elast. Winkel mit 3 Elementen, Last g & F = ,



■ 3.2 Berechnung der Statikloesung $F = -200$

$F1 = -200$

-200

■ Knotenverschiebungen $zFstat = Jy \cdot yF$, $yF = KFy^{-1} \cdot hFy$,

```
yF = Inverse[KFy] . hFy/.F->F1;
zFstat = Chop[yF];
If[nR>0, Do [zFstat = Insert[zFstat,0,Indr[[j]] ], {j,nR}]];
zFstat

{0, 0, 0, -1.5873×10-6, -0.0547618, -0.0824173,
 -3.1746×10-6, -0.121003, -0.0333403, 0.0119111, -0.121003, -0.0280086}

Print["Knoten k | u1 (m) | u2 (m) | theta3 (rad) "];
Do[Print[k, " | ", zFstat[[ (k-1)*nfk+1]],
 " | ", zFstat[[ (k-1)*nfk+2]], " | ", zFstat[[ (k-1)*nfk+3]]];
,
{k,
 nK}];

Knoten k | u1 (m) | u2 (m) | theta3 (rad)

1 | 0 | 0 | 0

2 | -1.5873×10-6 | -0.0547618 | -0.0824173

3 | -3.1746×10-6 | -0.121003 | -0.0333403

4 | 0.0119111 | -0.121003 | -0.0280086
```

■ Auflagerkraefte in der Bedeutung der Koordinaten nach Indr: $fbar = JFbar^T (KF \cdot zFstat - hF)$

```
fbar = KF . zFstat - hFsum; fbar = Chop[Expand[Delete[fbar,Partition[IndM,1]]]];MatrixForm[fbar]

( 200.
 164.81
 143.672 )
```

■ Schnittgröoessen je Element: $\sigma_{mae} = He \cdot epse = He \cdot Be \cdot Te \cdot zF$

```
sigmae = Table[0, {nE}, {2}];
Print["Element e | fle (N) | m3e (Nm) | me3(0) | me3(1) "];
Do[
sigmae[[e]] = Chop[Expand[He.Be.Transpose[TeT[[e]]].zFstat /.
 {le→Edata[[e, 5], Ae→Edata[[e, 2]], I33e→Edata[[e, 3]], Ee→Edata[[e, 1]]}]];
Print[e, " | ", sigmae[[e, 1]], " | ", sigmae[[e, 2]], " | ",
sigmae[[e, 2]] /. xi→0, " | ", Chop[sigmae[[e, 2]] /. xi→1 ]];
,
{e,
 nE}]

Element e | fle (N) | m3e (Nm) | me3(0) | me3(1)

1 | -200. | -139.257 + 138.323 xi | -139.257 | -0.934462

2 | -200. | -0.934462 + 85.349 xi | -0.934462 | 84.4145

3 | -29.431 | 80. - 80. xi | 80. | 0
```

■ Normalspannung = $\text{sigmae}[[1]]/\text{Ae}$ und Biegespannung = $\text{sigmae}[[2]] / \text{I33e} * \text{he}$

```
spannung = Table[0, {nE}, {2}];
Print["Element e | Normalspg (N/m^2) | Biegespg (N/m^2)"]
Do[
  spannung[[e, 1]] =
    Chop[sigmae[[e, 1]] / Ae /. {le → Edata[[e, 5]], Ae → Edata[[e, 2]], I33e → Edata[[e, 3]], Ee → Edata[[e, 1]]};
  spannung[[e, 2]] = Expand[sigmae[[e, 2]] / I33e * h1 /.
    {le → Edata[[e, 5]], Ae → Edata[[e, 2]], I33e → Edata[[e, 3]], h1 → Edata[[e, 7]]};
  Print[e, " | ", spannung[[e, 1]], " | ", spannung[[e, 2]]];
, {e, nE}]
```

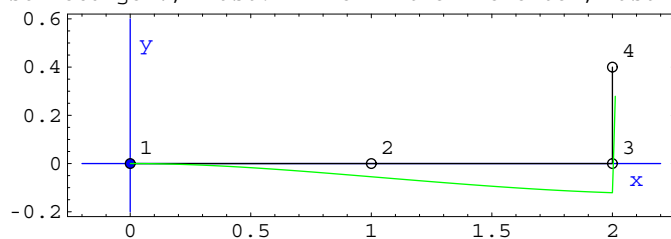
Element e	Normalspg (N/m ²)	Biegespg (N/m ²)
1	-111111.	$-1.03154 \times 10^8 + 1.02461 \times 10^8 \text{ xi}$
2	-111111.	$-692194. + 6.32215 \times 10^7 \text{ xi}$
3	-15490.	$5.3184 \times 10^7 - 5.3184 \times 10^7 \text{ xi}$

■ Grafiken für Verschiebung und Biegemoment

```
m3eSka = 1000;
fleSka = 500;
Ne3 = Join[Ne, {Table[0, {nFe}]}]; (* Nullzeile anhaengen *)
Ue = Table[0, {nE}, {3}];
plotUe = Table[0, {nE}];
plotf1 = Table[0, {nE}];
plotMe = Table[0, {nE}];
Do[Ue[[e]] = Transpose[Gammee[[e]] . Ne3 . Transpose[TeT[[e]]] . (zFstat + ZF) /. {le → Edata[[e, 5]], F → F1};
  plotUe[[e]] = ParametricPlot[{Ue[[e, 1]], Ue[[e, 2]]}, {xi, 0, 1},
    DisplayFunction → Identity, PlotStyle → {{RGBColor[0, 1, 0]}}];

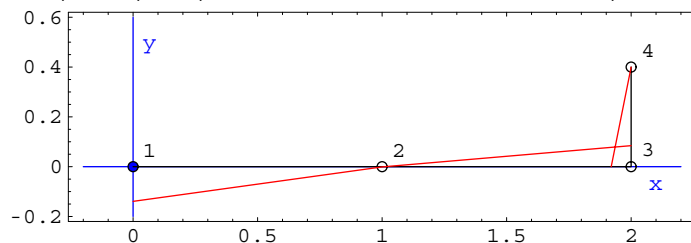
  Res = Transpose[Gammee[[e]] . {xi le,  $\frac{\text{sigmae}[[e, 2]]}{\text{m3eSka}}$ , 0} + RK[[e]] /. {le → Edata[[e, 5]], F → F1};
  plotMe[[e]] = ParametricPlot[{Res[[1]], Res[[2]]}, {xi, 0, 1},
    DisplayFunction → Identity, PlotStyle → {{RGBColor[1, 0, 0]}}];
  Res = Transpose[Gammee[[e]] . {xi le, sigmae[[e, 1]] / fleSka, 0} + RK[[e]] /. {le → Edata[[e, 5]], F → F1};
  plotf1[[e]] = ParametricPlot[{Res[[1]], Res[[2]]}, {xi, 0, 1},
    DisplayFunction → Identity, PlotStyle → {{RGBColor[1, 0, 0]}}];
, {e, nE}];
Show[PlStructure1, PlStructure2, PlStructure3, PlStructure4, plotUe, DisplayFunction → $DisplayFunction,
  PlotRange → All, PlotLabel → {"Knotenverschiebungen:", FEName, "Last g & F = ", F1}];
```

verschiebungen:, Elast. Winkel mit 3 Elementen, Last g & F =



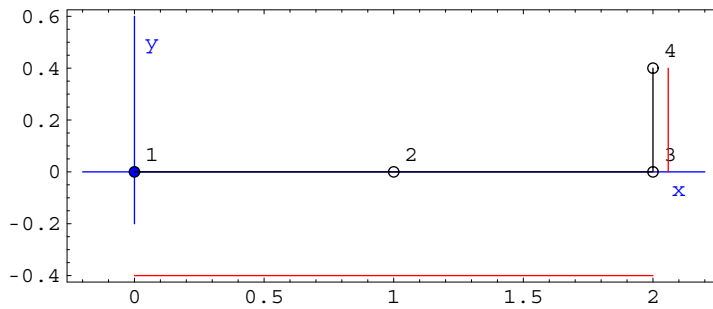
```
Show[PlStructure1, PlStructure2, PlStructure3, PlStructure4, plotMe,
  DisplayFunction → $DisplayFunction, PlotRange → All,
  PlotLabel → {"Biegemoment *", m3eSka, "Nm:", FEName, "Last g & F = ", F1}];
```

oment *, 1000, Nm:, Elast. Winkel mit 3 Elementen, Last g & F =



```
Show[PlStructure1,PlStructure2,PlStructure3,PlStructure4,plotf1,
  DisplayFunction->$DisplayFunction, PlotRange->All,
  PlotLabel->{"Normalkraft *",fleSka,"N:",FEName,"Last g & F = ",F1}];
```

kraft *, 500, N:, Elast. Winkel mit 3 Elementen, Last g & F =



■ 3.2 Berechnung der Statikloesung $F = +200$

$F1 = 200$

200

■ Knotenverschiebungen $zFstat = Jy \cdot yF$, $yF = KFy^{-1} \cdot hFy$,

```
yF = Inverse[KFy] . hFy/.F->F1;
zFstat = Chop[yF];
If[nR>0, Do [zFstat = Insert[zFstat,0,Indr[[j]] ], {j,nR}]];
zFstat

{0, 0, 0, 1.5873×10-6, -0.148824, -0.270542,
 3.1746×10-6, -0.497252, -0.40959, 0.165261, -0.497253, -0.414921}

Print["Knoten k | u1 (m) | u2 (m) | theta3 (rad) "];
Do[Print[k, " | ", zFstat[[ (k-1)*nfk+1]],
  " | ", zFstat[[ (k-1)*nfk+2]], " | ", zFstat[[ (k-1)*nfk+3]]],
{
{k,
nK}]];

```

Knoten k	u1 (m)	u2 (m)	theta3 (rad)
1	0	0	0
2	1.5873×10^{-6}	-0.148824	-0.270542
3	3.1746×10^{-6}	-0.497252	-0.40959
4	0.165261	-0.497253	-0.414921

■ Auflagerkraefte in der Bedeutung der Koordinaten nach Indr: $fbar = JFbar^T (KF \cdot zFstat - hF)$

```
fbar = KF . zFstat - hFsum; fbar = Chop[Expand[Delete[fbar,Partition[IndM,1]]]];MatrixForm[fbar]
```

$$\begin{pmatrix} -200. \\ 164.81 \\ 303.672 \end{pmatrix}$$

■ Schnittgrößen je Element: $\sigma_{\text{mae}} = H_e \cdot e_{\text{pse}} = H_e \cdot B_e \cdot T_e \cdot z_F$

```

sigmae = Table[0, {nE}, {2}];
Print["Element e | fle (N) | m3e (Nm) | me3(0) | me3(1) "]
Do[
  sigmae[[e]] = Chop[Expand[He.Be.Transpose[TeT[[e]]].zFstat /.
    {le → Edata[[e, 5]], Ae → Edata[[e, 2]], I33e → Edata[[e, 3]], Ee → Edata[[e, 1]]}]];
  Print[e, " | ", sigmae[[e, 1]], " | ", sigmae[[e, 2]], " | ",
    sigmae[[e, 2]] /. xi → 0, " | ", Chop[sigmae[[e, 2]] /. xi → 1] ];
,
{e,
 nE}]

```

Element e	fle (N)	m3e (Nm)	me3(0)	me3(1)
1	200.	-299.257 + 138.323 xi	-299.257	-160.934
2	200.	-160.934 + 85.349 xi	-160.934	-75.5855
3	-29.431	-80. + 80. xi	-80.	0

■ Normalspannung = $\sigma_{\text{mae}}[[1]]/A_e$ und Biegespannung = $\sigma_{\text{mae}}[[2]] / I_{33e} \cdot h_e$

```

spannung = Table[0, {nE}, {2}];
Print["Element e | Normalspg (N/m^2) | Biegespg (N/m^2) "]
Do[
  spannung[[e, 1]] =
    Chop[sigmae[[e, 1]]] / Ae /. {le → Edata[[e, 5]], Ae → Edata[[e, 2]], I33e → Edata[[e, 3]], Ee → Edata[[e, 1]]};
  spannung[[e, 2]] = Expand[sigmae[[e, 2]] / I33e * h1 /.
    {le → Edata[[e, 5]], Ae → Edata[[e, 2]], I33e → Edata[[e, 3]], h1 → Edata[[e, 7]]}];
  Print[e, " | ", spannung[[e, 1]], " | ", spannung[[e, 2]]];
, {e, nE}]

```

Element e	Normalspg (N/m^2)	Biegespg (N/m^2)
1	111111.	$-2.21672 \times 10^8 + 1.02461 \times 10^8 \text{ xi}$
2	111111.	$-1.19211 \times 10^8 + 6.32215 \times 10^7 \text{ xi}$
3	-15490.	$-5.3184 \times 10^7 + 5.3184 \times 10^7 \text{ xi}$

■ Grafiken für Verschiebung und Biegemoment

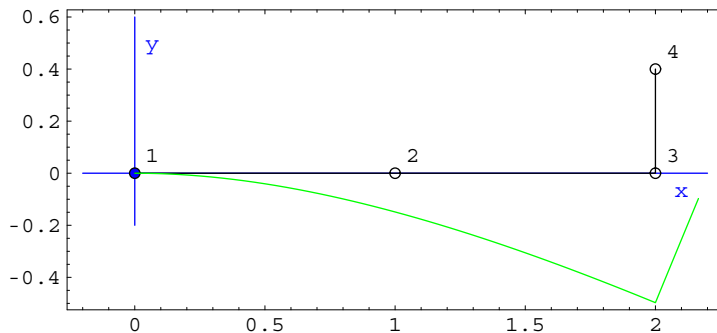
```

m3eSka = 1000;
fleSka = 500;
Ne3 = Join[Ne, {Table[0, {nFe}]}]; (* Nullzeile anhaengen *)
Ue = Table[0, {nE}, {3}];
plotUe = Table[0, {nE}];
plotf1 = Table[0, {nE}];
plotMe = Table[0, {nE}];
Do[Ue[[e]] = Transpose[Gammmae[[e]].Ne3.Transpose[TeT[[e]]].(zFstat + ZF) /. {le -> Edata[[e, 5]], F -> F1};
  plotUe[[e]] = ParametricPlot[{Ue[[e, 1]], Ue[[e, 2]]}, {xi, 0, 1},
    DisplayFunction -> Identity, PlotStyle -> {{RGBColor[0, 1, 0]}}];

Res = Transpose[Gammmae[[e]].{xi le,  $\frac{\text{sigmae}[[e, 2]]}{m3eSka}$ , 0} + RK[[e]] /. {le -> Edata[[e, 5]], F -> F1};
plotMe[[e]] = ParametricPlot[{Res[[1]], Res[[2]]}, {xi, 0, 1},
  DisplayFunction -> Identity, PlotStyle -> {{RGBColor[1, 0, 0]}}];
Res = Transpose[Gammmae[[e]].{xi le, sigmae[[e, 1]] / fleSka, 0} + RK[[e]] /. {le -> Edata[[e, 5]], F -> F1};
plotf1[[e]] = ParametricPlot[{Res[[1]], Res[[2]]}, {xi, 0, 1},
  DisplayFunction -> Identity, PlotStyle -> {{RGBColor[1, 0, 0]}}];
, {e, nE}];
Show[PlStructure1, PlStructure2, PlStructure3, PlStructure4, plotUe, DisplayFunction -> $DisplayFunction,
  PlotRange -> All, PlotLabel -> {"Knotenverschiebungen:", FEName, "Last g & F = ", F1}];

```

verschiebungen:, Elast. Winkel mit 3 Elementen, Last g & F =

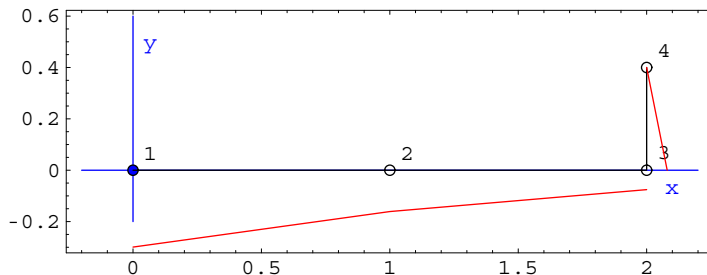


```

Show[PlStructure1, PlStructure2, PlStructure3, PlStructure4, plotMe,
  DisplayFunction -> $DisplayFunction, PlotRange -> All,
  PlotLabel -> {"Biegemoment *", m3eSka, "Nm:", FEName, "Last g & F = ", F1}];

```

oment *, 1000, Nm:, Elast. Winkel mit 3 Elementen, Last g & F =



```

Show[PlStructure1, PlStructure2, PlStructure3, PlStructure4, plotf1,
  DisplayFunction -> $DisplayFunction, PlotRange -> All,
  PlotLabel -> {"Normalkraft *", fleSka, "N:", FEName, "Last g & F = ", F1}];

```

lkraft *, 500, N:, Elast. Winkel mit 3 Elementen, Last g & F =

