Mode Participation Factor Computer (MPFC) Manual

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Overview – This manual gives an introduction to the use of the Mode Participation Factor Computer (MPFC) software. The MPFC is an independent add-on to multibody systems software such as SIMPACK that use SID data files to import flexible body data, enabling the user to visualise the mode shapes of flexible bodies and to optimise the selection of the mode shapes to be taken into account for dynamical simulations.

1. MATHEMATICAL BACKGROUND

It is assumed here that the reader is fairly familiar with the underlying mathematical principles of programmes such as SIMPACK. A comprehensible introduction can for example be found in [1] and [6]. The mode participation factors (MPF) method was, to the authors' knowledge, first published in [2]. A detailed description can also be found for example in [3], [4], [5] and [6]. However, a brief summary of MPF shall be given as follows.

Using dots to denote derivations with respect to time the equations of motion of a flexible body can be written as

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{D}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{f} + \mathbf{G}^{\mathsf{T}}\boldsymbol{\lambda} \tag{1a}$$

subject to kinematic constraints $\qquad \mathbf{g} = \mathbf{0} \;$ and $\; \mathbf{G} = \dfrac{\partial \mathbf{g}}{\partial \mathbf{q}}$ (1b, 1c)

where **M** is the body generalised mass matrix, **D** a matrix containing damping coefficients, **K** the body generalised stiffness matrix, **f** the vector of generalised applied (external) forces and **q** the vector of body degrees of freedom (6 + *n* rigid and elastic degrees of freedom). $\boldsymbol{\lambda}$ is a vector of Lagrange's multipliers and **G** is a matrix obtained by deriving the vector of kinematic constraints **g** with respect to **q**. Note that the use of **g** is such that every entry shall be zero, thus **g = 0**.
For many simulations it is justified to assume that

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{D}\dot{\mathbf{q}} \ll \mathbf{K}\mathbf{q} \tag{2}$$

for many points in time. This is for example true for slowly moving flexible bodies, for bodies with only little structural damping (usually 1%), for flexible bodies under static or slowly changing loads and so on. With the assumption expressed by equation (2), equation (1a) simplifies to give

$$\mathbf{K}\mathbf{q} = \mathbf{f} + \mathbf{G}^{\mathsf{T}}\boldsymbol{\lambda} \tag{3}$$

In order to solve (3), we let

$$\begin{bmatrix} \mathbf{K} & -\mathbf{G}^{\mathsf{T}} \\ -\mathbf{G} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{q} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{f} \\ \mathbf{0} \end{bmatrix} \tag{4}$$

The horizontal and vertical lines in equation (4) indicate how the system matrices are made up of the matrices introduced before. Knowing **K**, **G** (and its transpose) and **f**, equation (4) can be solved for the vector of modal weighing coefficients **q** and Lagrange's mulitpliers $\boldsymbol{\lambda}$. The latter represent the forces associated with the kinematic constraints of the **q** system itself, that is the joint (internal) forces. Note that equation (4) is non-singular and can thus be solved only if the constraints expressed with the constraint matrix **G** are such that no rigid body motion is possible.

The mode participation factors (MPF) are computed using the modal weighing coefficients as follows, where $i = 1$, $2, ..., n_e$ (number of elastic degrees of freedom or modes). The i-th modal MPF, MPF_i^q, is the absolute value of the i-th weighing coefficient q_i in vector $\mathbf{q}$ divided by the sum of the absolute value of all weighing coefficients.

$$MPF_i^q = \frac{|q_i|}{\sum_i |q_i|} \times 100\% \qquad (5)$$

The total elastic deflection $\mathbf{u}$ of the body at hand at position (x, y, z) is the sum of all modal weighing coefficients times the associated modal displacements at (x, y, z), or

$$\mathbf{u}(x,y,z) = \sum_i \mathbf{\Phi}_i(x,y,z)\, q_i \qquad (6)$$

The elastic displacement vector $\mathbf{u}$ is a 6×1 vector containing the three translations in x, y and z and the three rotations about x, y and z. The modal displacement matrices $\mathbf{\Phi}_i$ are the so-called mode shapes.
In MBS programmes, the mode shapes matrices $\mathbf{\Phi}_i$ only contain the information of the elastic deflections of points upon the body that were explicitly selected beforehand. Such points are called "markers" or "nodes". For example, if we want to model an elastic beam that connects some other parts of a multibody system, we only need one node or marker at each end of the beam; the elastic deflections of all other points upon the beam are irrelevant.
If we let subscript α be the six directions of elastic deflections (three translations and three rotations) and k any node upon the flexible body, equation (6) can be written more precisely

$$u_\alpha^k(x,y,z) = \sum_i \Phi_{\alpha,i}^k(x,y,z)\, q_i \qquad (7)$$

In general, all $\Phi_{\alpha,i}^k$ are different. This means that for each node k at position (x, y, z) upon the body and again for each of the six elastic deflections α at (x, y, z) the sum expressed in equation (7) and its individual terms are different. We get a total of $n_e \times 6$ nodal MPF for each node k at position (x, y, z) upon the body, the $MPF_{\alpha,i}^k$ as

$$MPF_{\alpha,i}^k = \frac{\left|\Phi_{\alpha,i}^k(x,y,z)\, q_i\right|}{\sum_i \left|\Phi_{\alpha,i}^k(x,y,z)\, q_i\right|} \times 100\% \qquad (8)$$

2. INSTALLING AND DEINSTALLING THE MPFC

Necessary files: The MPFC is distributed as an install package for Windows operating systems. The following files should be included:

- *instmsiw.exe*
- *instmsi.exe*
- *DistFile.cab*
- *setup.exe*
- *setup.ini*
- *SID_and_Modes.msi*

These files together should have a size of about 5.5 MB. If they are compressed into one single ZIP archive the archive should have a size of about 5 MB. If the size on your computer differs significantly, the package may be corrupted or otherwise modified.

System Requirements: Although no real test was made on different computers, it is recommended that your system has a clock speed of at least 300 MHz, together with a memory of at least 256 MB RAM. Also it is assumed that your system has a video card according to today's standards.

Display Settings: The MPFC is best used with a display resolution setting of 1280 × 1024 pixels with at least high colour (16 bit). To change your display settings, right-click on an empty space on your desktop and choose *Properties*. In the appearing window click on the "Settings" tab.

Running the Install File: To start the installation, simply run the *setup.exe*. This will install the run time machine, the graphical user interface and some dynamic link libraries (*.dll) on your computer. You will probably need administrator access to the machine you are planning to install the MPFC on.
After having successfully installed the MPFC on your computer, you should have an additional folder

"SID_and_Modes" in the *Start → Programs* menu. In this folder you find the *SID_and_Modes.exe*.

Deinstalling the MPFC: To deinstall the MPFC from your machine use the Windows *Start* button in the task bar and go to *Settings → Control Panel → Software*. Choose the MPFC (named "SID_and_Modes" in the *Software* window) and click the *Delete* button. This will deinstall all MPFC files from your computer. If some files and / or folders remain you can delete them manually.

3. STARTING THE MPFC AND LOADING A SID FILE

In the Windows *Start → Programs* menu, go to the folder *SID_and_Modes* and start the *SID_and_Modes.exe*. This will open the following window on your desktop.
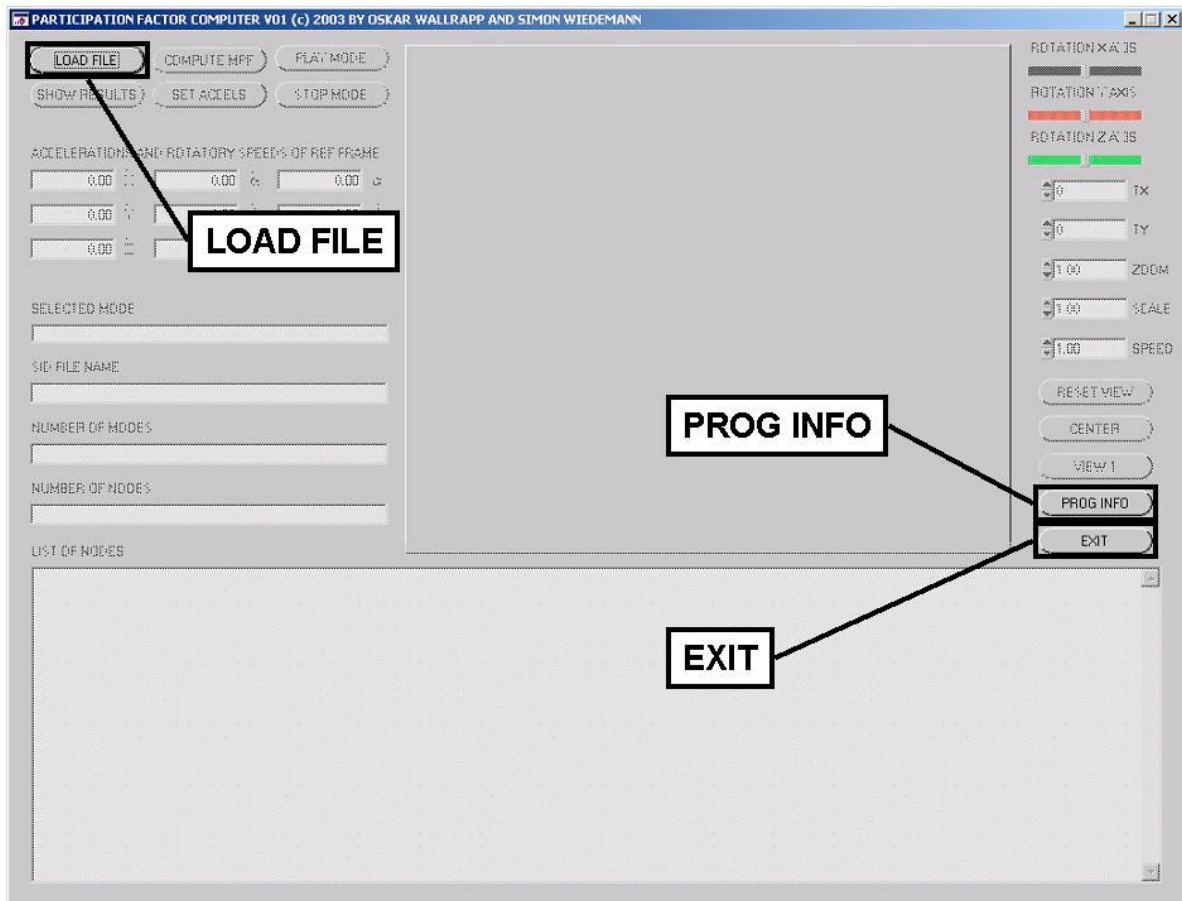

As you can see, all but three buttons are greyed out:

PROG INFO   Click on this button to get information about the programme. You will get the names and email addresses of the authors, a reminder in view of intellectual property and the current software version.

EXIT   At any time, click on this button to properly shut down the application. This will unload any *.SID files, free the memory and end the processor thread.

LOAD FILE   Click this button to browse for a *.SID file as used by SIMPACK. This will bring up the window shown on the right. Note that by default, the filter is "*.SID_FEM", as can be seen in the above screenshot. If you have a SID file with another file extension, change the filter to "All files (*.*)". Otherwise use this file browser to find the SID file you want to analyse or optimise for use in dynamical simulations in SIMPACK. Highlight the file and press the *LOAD* button or press *CANCEL*.
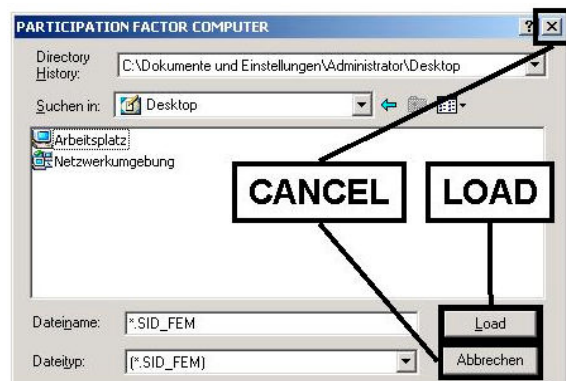
4. DISPLAY OPTIONS OF THE MPFC

If you load a valid SID file and the MPFC can read all necessary data, your main window changes to the active state, along with most of the controls and command buttons. If the file cannot be identified as a valid SID file, the MPFC will display a warning and remain in the inactive or startup state.
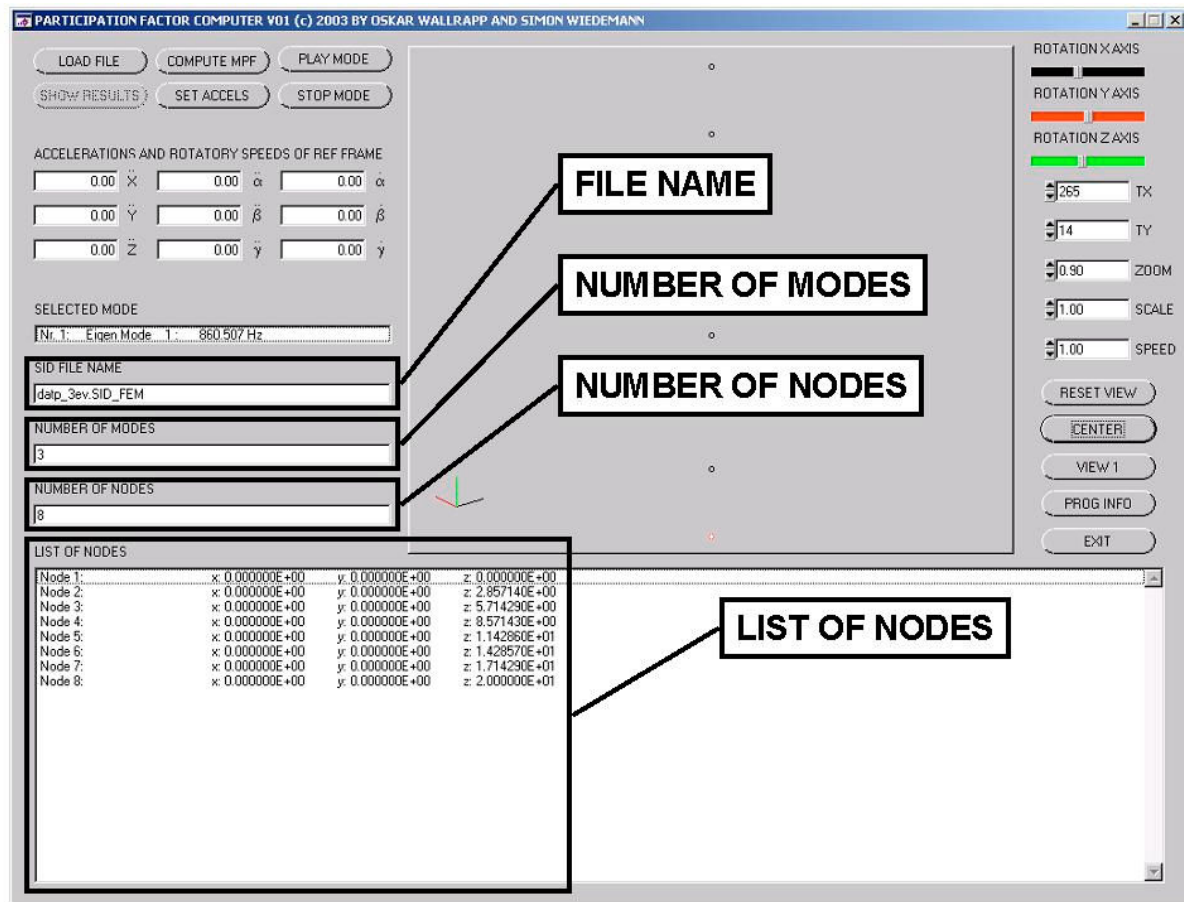
The MPFC will display the name of the SID file you loaded, the number of modes, the number of nodes contained in the SID file and a list of all nodes. This list consists of the names of the nodes as saved in the SID file and the position of the nodes in x, y and z direction, relative to the body reference frame. The order of the nodes in the list is equal to the order as they are saved in the SID file and has nothing to do with the names or the position of the nodes.

The MPFC provides an area to display the flexible body defined by the loaded SID file. More precisely, the nodes for which data are stored in the SID file and which appear in the list of nodes, are represented by a small circle each, and the origin of the body reference frame is represented by a white diamond. The more nodes your flexible structure consists of, the better your imagination of the body and its modes will be. However, the computations and their precision are not at all affected by the number of nodes.

A number of display functions are implemented into the MPFC so that you can comfortably analyse and observe the behaviour of your flexible body.

ROTATION   Use the three rotation sliders in the right upper corner of your main window to rotate the body shown in the mode display area. Note that the rotations take place about the body fixed x, y and z axes that are shown in the left bottom corner of the display area. These three axes will move accordingly when you change the position of one of the three rotation sliders. The three axes have three different colours that match the colours of the sliders. Thus if for example you want to rotate the body about the red axis as shown in the left bottom corner of the display area, use the red slider (y axis).

SHIFTING   Use the two translation number boxes *TX* and *TY* below the rotation sliders to move the representation of your flexible body in the display area left and right (x direction) and up and down (y direction). **Caution: the x and y direction mentioned here are only the display left-right and up-down! They have nothing to do with the body reference frame x, y and z!** The numbers in the boxes correspond to pixels in the display. You can use the arrows to increment / decrement the value or enter another value in the number box. For example, if you would like to

see the bottom part of your structure that is momentarily out of the display area at the bottom, increment the value for translation along *y* until you can see the missing part.

ZOOM Use the zoom number box to zoom in and out of the display area and your flexible body. This function is valuable when you have a large structure and many nodes, so that by zooming into the display area and maybe rotating and shifting the body a bit, you can concentrate on certain areas of interest regarding the body under observation.

RESET VIEW This function will reset all rotational and translational settings made by the user. This means that the origin of the body frame of reference (position 0, 0, 0) is in the left bottom corner of the display window. Additionally, the view (see TOGGLE views) is set to the *x-y*-plane. Maybe you cannot immediately find your body after resetting the view, but that is why you have the *TOGGLE* button and the *CENTER* button. The zoom setting is not affected by resetting the view.

CENTER This button will cause the body to be centred in the display area. This means that the translations *TX* and *TY* are calculated and set so that the centre of the body coincides with the centre of the display area. Since the rotation and zoom setting are not affected, you might not see all nodes of your flexible body. Use the zoom function to make sure you can see all of your body.

TOGGLE Use the *VIEW* button to toggle between four default views that are provided by the MPFC. The four default views are:
1: some spatial view as if the body is seen from a certain angle (45°)
2: body as seen in its *y-z*-plane
3: body as seen in its *x-z*-plane
4: body as seen in its *x-y*-plane
Toggling the view will reset all rotational settings made before and set the default rotations according to the corresponding view. In parallel, the body is centred in the display area in each of the default views, which means that translational settings are also overwritten. The zoom setting is not affected by toggling views.

After loading a new SID file, the display view is VIEW1, the zoom is set to 1 and the body is centred. Thus independent of the size of the structure as expressed by the node position co-ordinates in the list of nodes, you should always have a centred body that is completely visible within the display area.
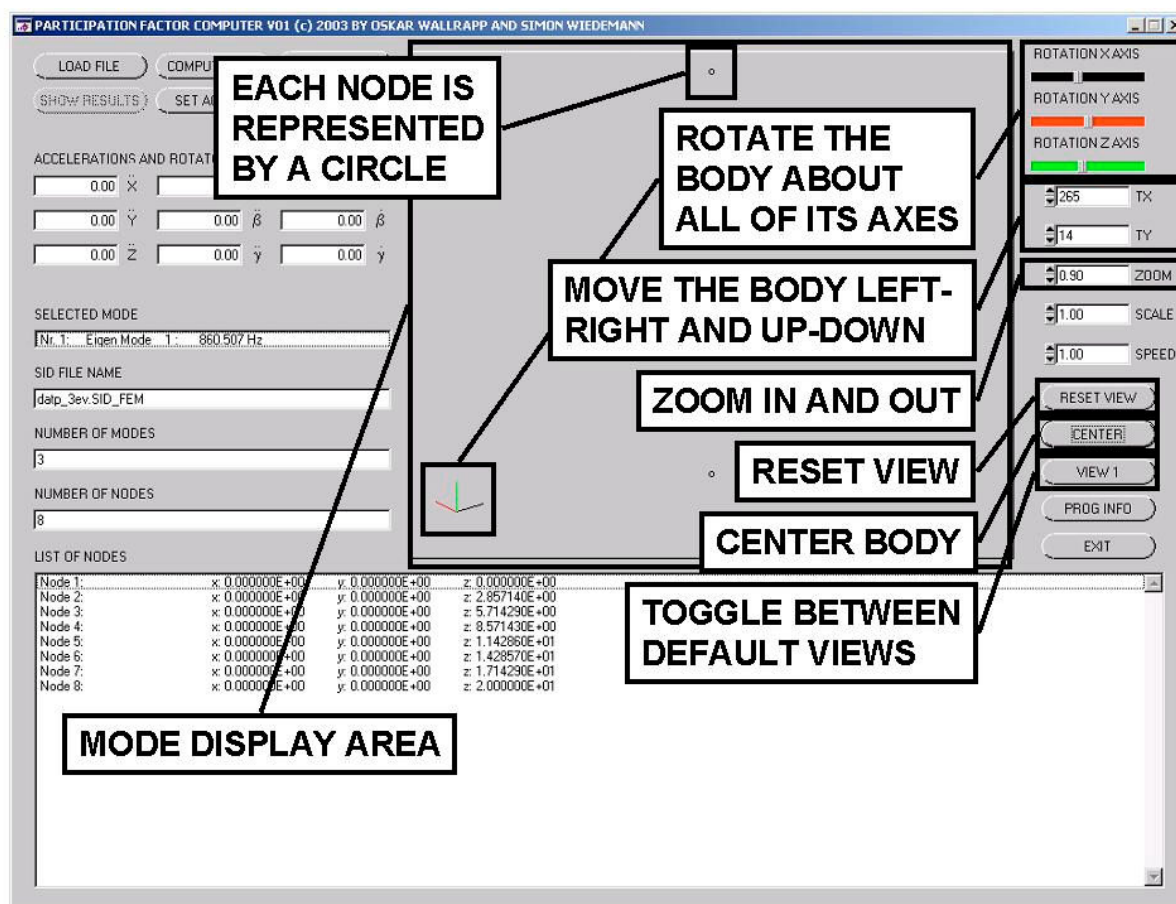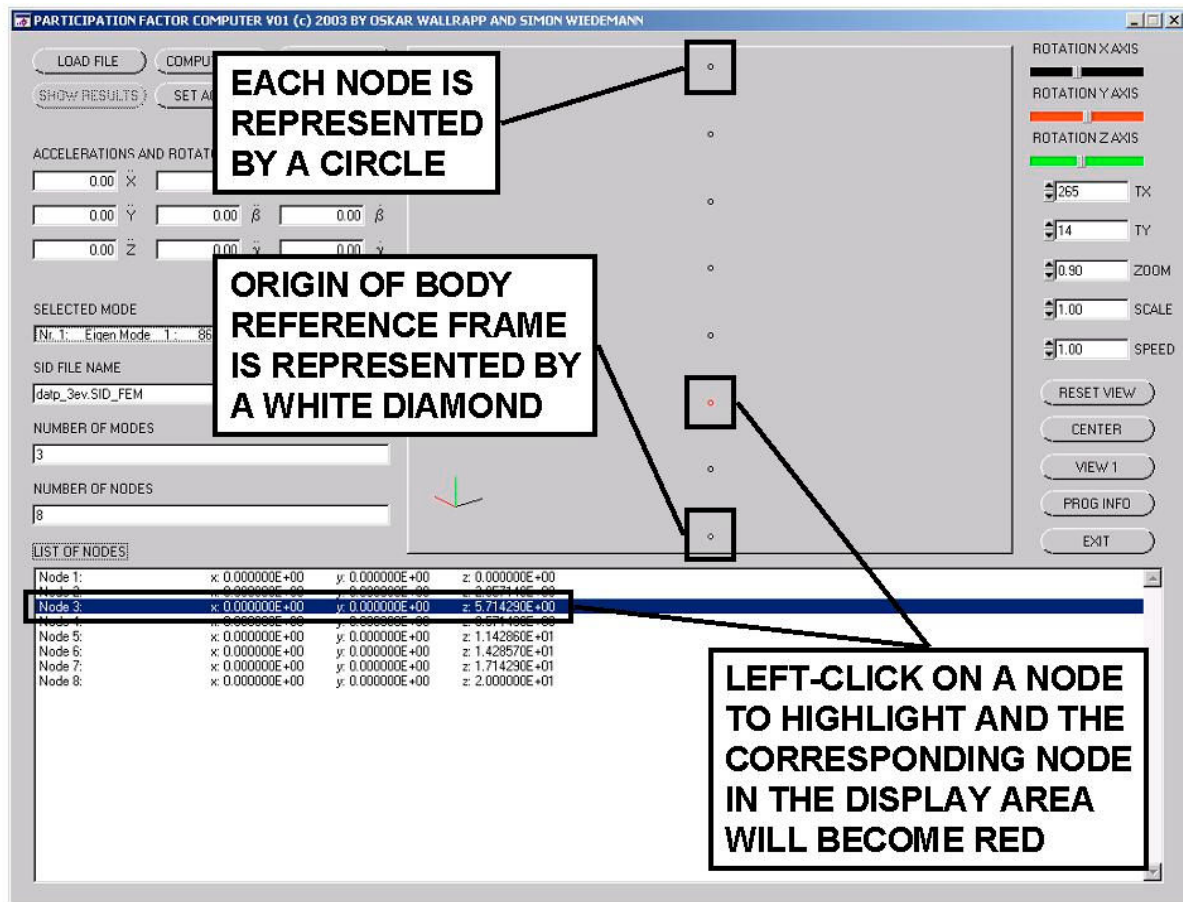
We already know that each node of the flexible structure is represented by a small circle in the display area. If you want to quickly find a special node in the display, simply left-click once on the node in your list of nodes in the lower part of the main window. Now the corresponding node or circle in the display becomes red, by contrast to all other nodes that are normally displayed in black colour. The origin of the body, that is the position (0, 0, 0) is always displayed as a small white diamond, even if there is no node with the position (0, 0, 0).

What more display options do you have? One of the main benefits of the MPFC is its capability to display the individual vibration modes. You have the option shown in the following figure. For the mode visualisation and analysis you have basically these five control elements:

PLAY MODE When you left-click this button once, the mode shown in the *SELECTED MODE* text box is played endlessly in the display area. This gives you the feeling as though your flexible structure is really vibrating back and forth in the respective mode. Note that all control elements remain sensitive so that you can always play a mode just for your pleasure and go on with setting the boundary conditions and compute the MPF. Note also that playing a mode has no effect on the other display settings mentioned before so that using zoom, translation and rotation you can observe and analyse every part of your flexible body in detail during vibration.

STOP MODE This button stops the endless playing of the mode shape and the body is displayed as usual without any elastic deflections. The user translation, rotation and zoom settings are effective.

SELECTION The text box *SELECTED MODE* shows the mode shape currently selected, that is its number in the order of the mode shapes in the SID file, its name according to the SID file and the associated eigenfrequency, also according to the SID file. After loading a SID file, the first mode shape found in the SID file is shown by default in the *SELECTED MODE* text box. To select another mode left-click on the *SELECTED MODE* text box once to highlight it. Then use the up and down arrow keys on your keyboard to go through the list of mode shapes in the order as they are stored in the SID file. The selected mode shape is displayed in the text box.

MODE SCALE Use this number box to change the scale of the mode shape after the play button was pressed. Scale means that by entering a smaller or larger figure in the number box, the elastic deflections shown when a mode is played can be made smaller or larger. In other words, the larger the figure in the *SCALE* number box, the larger the amplitude of vibration of your structure during playing a certain mode shape.

MODE SPEED Use this number box to set the speed with which the mode shape is played. Note that due to limits in the refresh frequency of the display area, changing the value of the *SPEED* number box beyond some certain limit might not lead to any visible effects. The larger the number, the quicker the mode shape vibration in the display area. **CAUTION**: on some machines, due to some miraculous reasons, setting the speed to zero causes the programme to hang. So always enter at least a minimum value of 0.01.
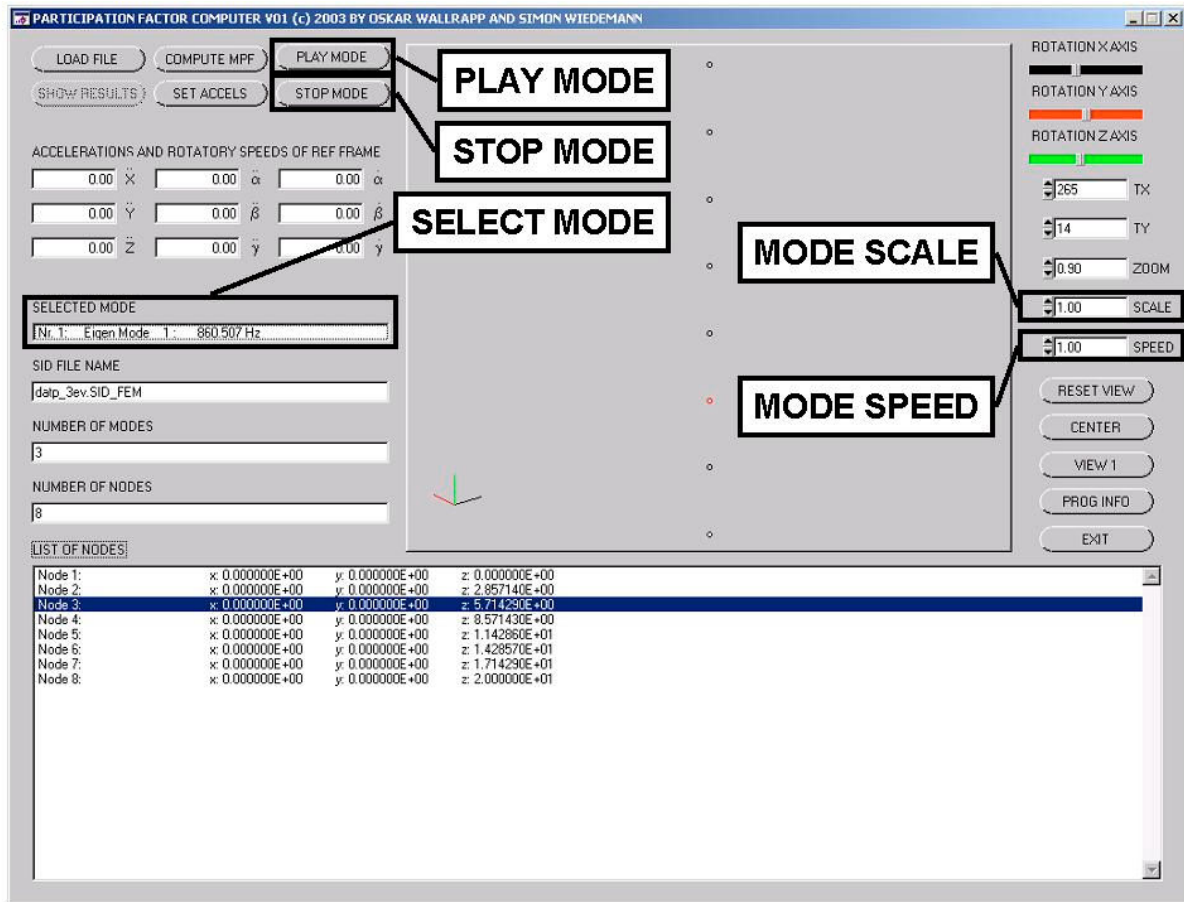

Now you know all display possibilities the MPFC offers you. In the next steps you learn how to define the boundary conditions for computing the mode participation factors, MPF.

5. SETTING BOUNDARY CONDITIONS FOR THE COMPUTATION OF THE MPF

The boundary conditions for a flexible body consist of two parts, kinematic constraints and loads (applied forces). The kinematic constraints you enter will be taken by the MPFC to compute the constraint matrix **G** (and its transpose $\mathbf{G}^T$) for equation (4), and the vector of generalised forces **f** is made up of applied forces and body reference frame accelerations that lead to forces due to rotary and translatory mass inertia.

In order to set nodal boundary conditions you first have to highlight the node you want to apply boundary conditions to in the list of nodes by left-clicking it once. After the node of your choice is highlighted, right-click it as shown in the following figure.
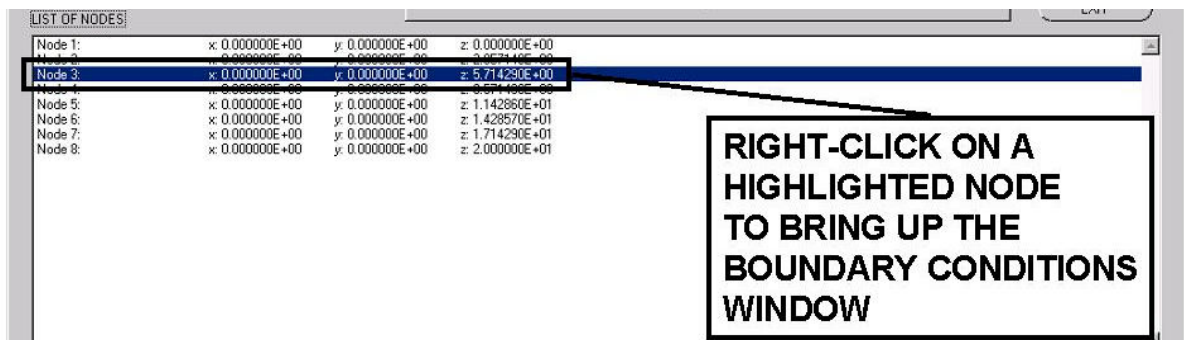
This will bring up the following boundary conditions window.

You can apply three translatory forces in [N] and three rotatory forces (torques) in [Nm].

The six boxes can be ticked individually. Each ticked box adds the corresponding constraint along or about one of the three axes.

Note that it makes no sense to restrain say, the translation along *x* and at the same time apply a force along *x*.

Click the *RESET* button to set all applied forces and torques back to zero and to clear the tick boxes of all kinematic constraints.

Pressing the *CANCEL* button will close the boundary condition window without effect.

Click the *OK* button to close the boundary condition window and to display the entered figures in the list of nodes in the main window.

An example of some boundary conditions is shown next.
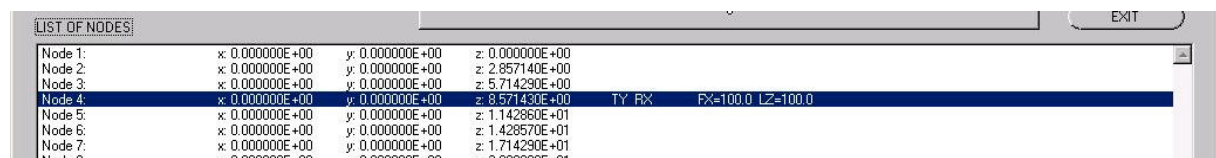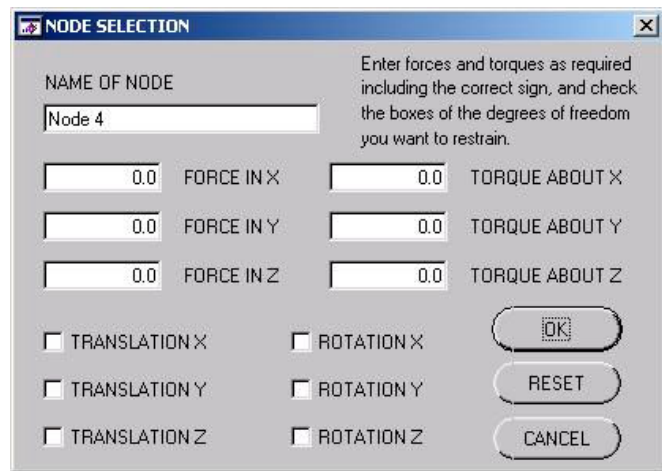
The above figure shows the lower part of the MPFC main window with the list of nodes. As can be seen, boundary conditions were defined for the node with the name "Node 4", and these boundary conditions are displayed in the list of nodes. The meaning of the abbreviations is as follows.

TX / TY / TZ   kinematic constraint; the translation along *x* and / or *y* and / or *z* is locked
RX / RY / RZ   kinematic constraint; the rotation about *x* and / or *y* and / or *z* is locked
FX / FY / FZ   applied force; a force along *x* and / or *y* and / or *z* is applied with the given magnitude
LX / LY / LZ   applied torque; a torque about *x* and / or *y* and / or *z* is applied with the given magnitude

Thus in the above example the node named "Node 4" is locked along the *y* axis (TY) and about the *x* axis (RX), and a force of 100 N is applied along *x* (FX=100.0) and a torque of 100 Nm is applied about *z* (LZ=100.0).

Note that according to what has been said a few times before, the two kinematic constraints in this example are not sufficient to restrict all rigid body motions. Therefore, more constraints would have to be applied at other nodes.
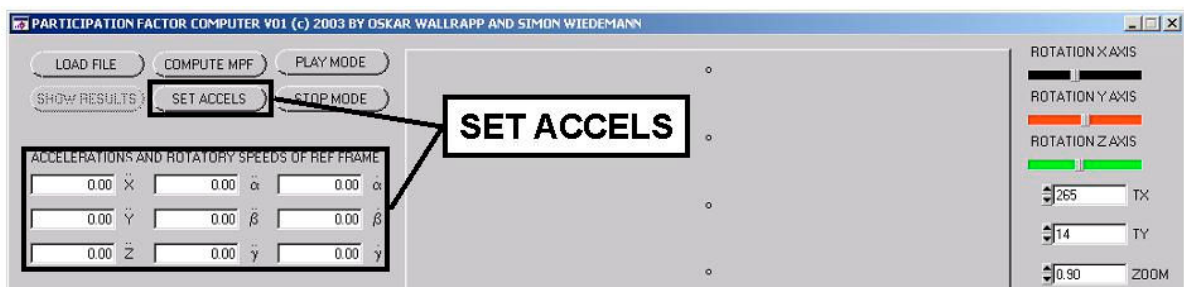
If you left-click the button *SET ACCELS* shown in the above figure, the window on the right will appear (note that the number boxes in the main panel are only for display, not for entering other figures – you have to use the *SET ACCELS* button). This new window is pretty self-explaining. You see that you can enter translational accelerations along the three body axes (X, Y, Z with two dots), rotational accelerations about the three body axes (α, β, γ with two dots), and also constant rotary velocities about the three body axes (α, β, γ with one dot).

Note that the translatory and rotatory accelerations are entered in [m/s^2] and [rad/s^2],

respectively, the rotatory velocities in [rad/s]. For example, if you enter a translatory acceleration along *y* of -9.81, you simulate the flexible body under gravity. The reason why you can enter also rotational velocities is due to the
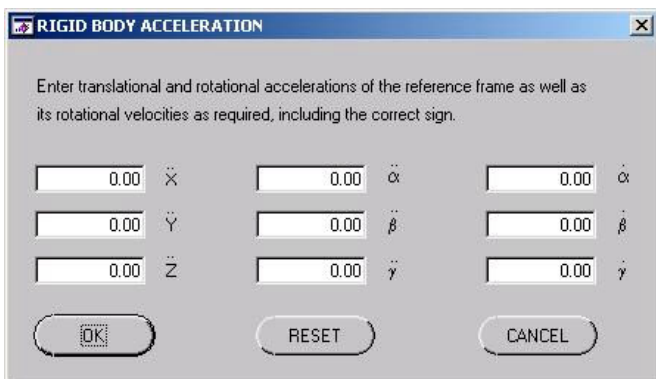
fact that even constant rotations cause centrifugal accelerations.

Click the *RESET* button to set all accelerations back to the zero. Pressing the *OK* button will close the window and display the entered figures in the *ACCELERATIONS AND ROTATORY SPEEDS OF REF FRAME* number boxes of the main window. Pressing the *CANCEL* button will close the acceleration window without effect.

As was said before, by using the boundary conditions window for one or more nodes you define the constraint matrix **G** (and its transpose **G**^{T}) and the vector of generalised forces **f** in equation (4). Now since **K** is known from the SID file equation (4) can be solved for the modal weighing coefficients **q** and the Lagrangian multipliers λ (of which no further use is made after computation).
Note however that as was mentioned repeatedly before, the kinematic constraints must be chosen so that no rigid body motion is possible, otherwise equation (4) cannot be solved.

6. COMPUTING AND DISPLAYING THE MPF

After having defined the boundary conditions you can finally compute the MPF. This is done using the *COMPUTE MPF* button shown in the following figure.
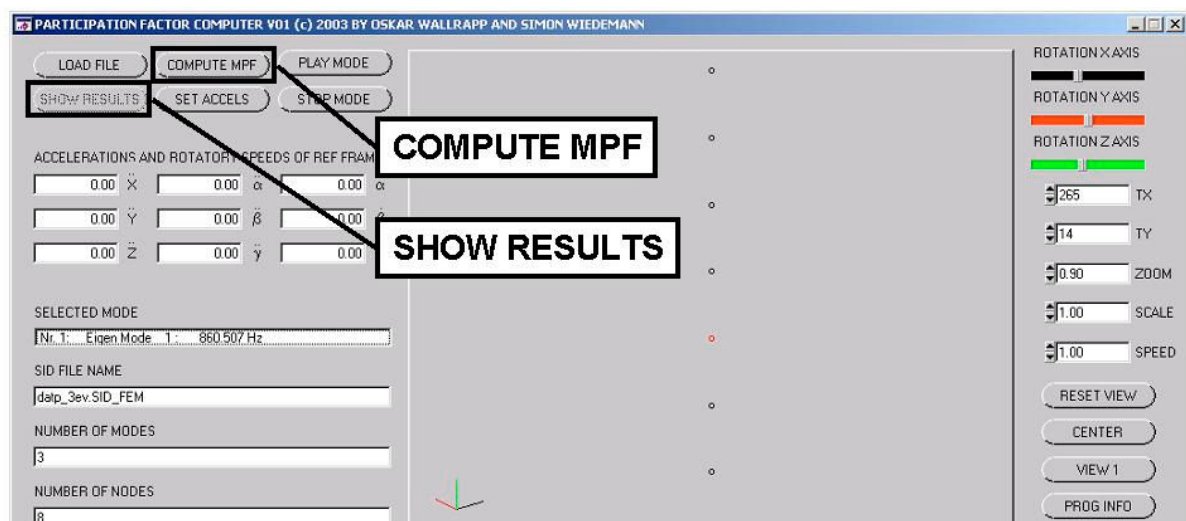
Remember that your kinematic constraints must make sure that the flexible body at hand cannot perform any rigid body motion. If this is not the case, the MPFC will display the warning shown on the right, remembering you to modify the constraints and / or increase their number. After this warning the MPFC will display another message box informing you that the computation of the weighing coefficients **q** in equation (4) was not successful.
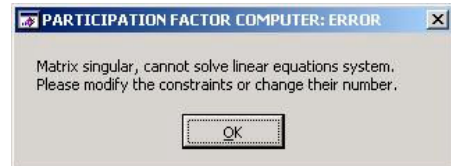

Sometimes equation (4) can be solved, but the pivot element is very small, giving a so-called "ill-conditioned" matrix. If the MPFC recognises such an ill-conditioned matrix, it will display a warning together with the advice to try and modify the boundary conditions a little bit. However, the computation of the weighing coefficients is nonetheless successful.

If the boundary conditions are appropriate and if the linear equation system (4) can be solved, the weighing coefficients **q** are computed and you will get the message shown on the right.
At the same time the *SHOW RESULTS* button will no longer be greyed out and become sensitive.

When you left-click the *SHOW RESULTS* button, the final MPF expressed in equations (5) through (8) are computed. Note that it is possible to compute the weighing coefficients without having defined any applied forces or inertia forces due to accelerations. This will lead to a trivial all zero solution vector and the MPFC will give you a respective warning as shown on the right. Confirming this message by clicking the *OK* button will bring you back to the MPFC main window.
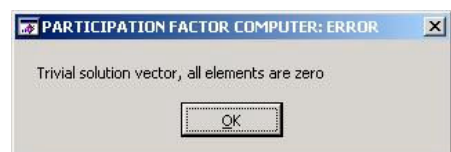
Otherwise if the solution vector is not trivially all zero the following results window will appear.
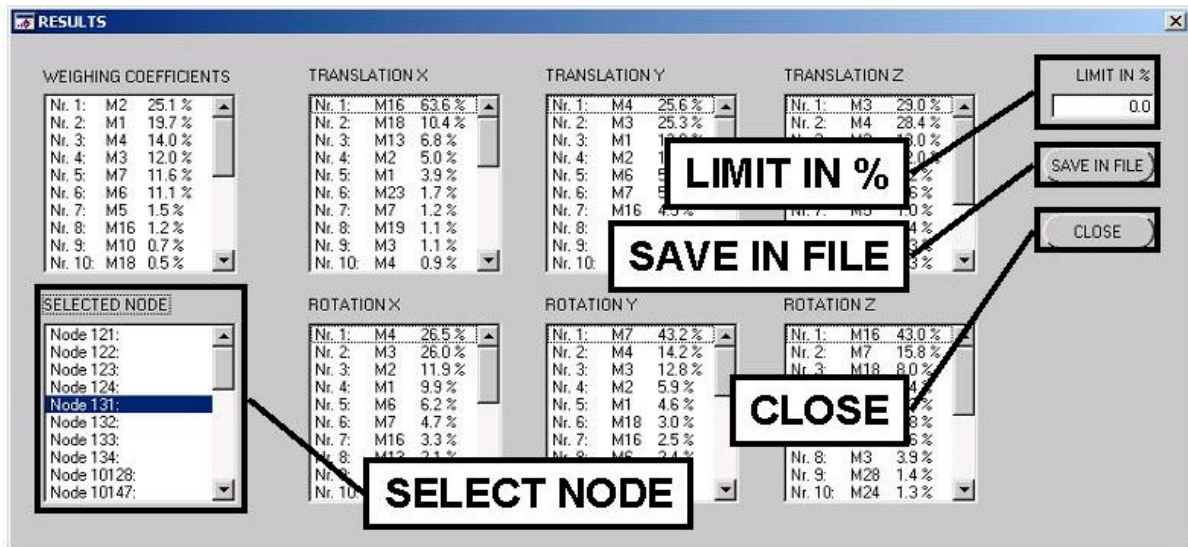
In the upper left corner you see the *WEIGHING COEFFICIENTS* results. This is the solution of equation (5) for the weighing coefficients q_i in vector **q**, which in connection with equation (5) we called the *modal* MPF. This solution is interesting in view of the excitation of the various eigenmodes for the defined boundary conditions (consisting, as was said before, of kinematic constraints and loads).

In the above example, the mode excited the most is mode number 2, participating with 25.1 % in the overall solution ("Nr. 1: M2 25.1 %"). The eigenmode number 4 is the third mode in the order of their excitation ("Nr. 3: M4 14.0 %") and so on.

However, as was said in connection with equation (4) and the following equations, to get the solution with regard to a special node and also with regard to a special degree of motional freedom (that is, one of the three basic translation or one of the three basic rotations), we have to compute what we called before the *nodal* MPF as expressed with equation (8).

This solution is given in the 6 text boxes called *TRANSLATION X*, *TRANSLATION Y*, *TRANSLATION Z*, *ROTATION X*, *ROTATION Y* and *ROTATION Z*. Equation (8) and the associated text indicate that the nodal MPF are different for every individual node of the flexible body and that the equation must therefore be computed for each node. The node for which equation (8) shall be solved can be selected and highlighted in the *SELECT NODE* box by left-clicking the respective node once.

In the above example, for the node named "Node 131" in the loaded SID file and regarding the flexible translation in the body *x* direction, then mode number 16 is the most excited, participating with 63.6 % in the total deflection ("Nr. 1: M16 63.6 %"). Regarding the rotation about the body *y* axis, mode number 7 is the most excited, making for 43.2 % of the total elastic rotary deflection about *y* ("Nr. 1: M7 43.2 %") and so on. Note that if we highlighted any other node then, in general, mode number 7 will not be the most excited regarding the rotational elastic deflection about the body *y* axis!

What does this mean for the use of the MPFC? In practice, you are usually interested in the motion of only a few nodes, and then probably only with regard to say, 2 or 3 directions (or rotations). Thus for example if you have a node which you want to connect a force element or another body to, you would highlight that node in the results window shown above and would consider the degrees of freedom you are interested in. Then you pick the modes in the order they appear in the above results window until say, their share in the overall solution is less than 2 %. This leaves you with for example 6 modes which you should choose for dynamical simulation in SIMPACK.

Although in reality the MPF are computed for quasi-static load cases, only a few typical load cases usually suffice to roughly describe what happens to your flexible body during simulation. Thus if for example you have 3 or 4 typical load cases, you would compute the nodal MPF you are interested in for these few cases and then make a sensible selection of modes so that each of the load cases is fairly addressed.

Now let us consider the last three user controls. In the *LIMIT IN %* box you can enter an arbitrary minimum share percentage for each mode. Thus for example if you only want to see the modes that have a share of at least 1 % in the overall solution, you enter "1.0" in the number box. All modes in all the result boxes with a share of less than 1 % will be deleted so that you cannot see them any more. The modes that remain are the ones you are interested in. Note that the limit affect all seven result boxes for the modal and nodal MPF.

The *SAVE IN FILE* button prompts you for a name and path of a result file that is created by the MPFC. The file is tabulator separated and has a standardised form so that other programmes may later read such a result file and make further use of the results. The file contains all necessary information such as date, time, name of the SID file, boundary conditions, share percentage limit and so on. This is also good for automatically documenting what you have done.

Finally, the *CLOSE* button closes the results window and brings you back to the MPFC main window.

Congratulations, you have made it through the manual! Now try the software and send us your impressions if you want to. You can find the names and email addresses of the authors by clicking the *PROG INFO* button in the main MPFC window (see also page 3 of this manual).

7. REFERENCES

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