■ Uebung 3.5: Gleichungen der Verzerrungen ϵ und Spannungen σ und das Materialgesetz - 3D

```
ϵ = {
  {ϵ1},
  {ϵ2},
  {ϵ3},
  {γ12},
  {γ23},
  {γ31}
 }
```

```
{{ϵ1}, {ϵ2}, {ϵ3}, {γ12}, {γ23}, {γ31}}
```

```
σ = {
  {σ11},
  {σ22},
  {σ33},
  {τ12},
  {τ23},
  {τ31}
 }
```

```
{{σ11}, {σ22}, {σ33}, {τ12}, {τ23}, {τ31}}
```

```
LOperator = {
  {diff1, 0, 0},
  {0, diff2, 0},
  {0, 0, diff3},
  {diff2, diff1, 0},
  {0, diff3, diff2},
  {diff3, 0, diff1}
 }; MatrixForm[LOperator]
```

$$\begin{pmatrix} \text{diff1} & 0 & 0 \\ 0 & \text{diff2} & 0 \\ 0 & 0 & \text{diff3} \\ \text{diff2} & \text{diff1} & 0 \\ 0 & \text{diff3} & \text{diff2} \\ \text{diff3} & 0 & \text{diff1} \end{pmatrix}$$

```
HMat = E0 / (1 + μ) / (1 - 2 μ) *
  {
   {1 - μ, μ, μ, 0, 0, 0},
   {μ, 1 - μ, μ, 0, 0, 0},
   {μ, μ, 1 - μ, 0, 0, 0},
   {0, 0, 0, (1 - 2 μ) / 2, 0, 0},
   {0, 0, 0, 0, (1 - 2 μ) / 2, 0},
   {0, 0, 0, 0, 0, (1 - 2 μ) / 2}
  }; HMat // MatrixForm
```

$$\begin{pmatrix} \frac{E0\,(1-\mu)}{(1-2\,\mu)\,(1+\mu)} & \frac{E0\,\mu}{(1-2\,\mu)\,(1+\mu)} & \frac{E0\,\mu}{(1-2\,\mu)\,(1+\mu)} & 0 & 0 & 0 \\ \frac{E0\,\mu}{(1-2\,\mu)\,(1+\mu)} & \frac{E0\,(1-\mu)}{(1-2\,\mu)\,(1+\mu)} & \frac{E0\,\mu}{(1-2\,\mu)\,(1+\mu)} & 0 & 0 & 0 \\ \frac{E0\,\mu}{(1-2\,\mu)\,(1+\mu)} & \frac{E0\,\mu}{(1-2\,\mu)\,(1+\mu)} & \frac{E0\,(1-\mu)}{(1-2\,\mu)\,(1+\mu)} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{E0}{2\,(1+\mu)} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{E0}{2\,(1+\mu)} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{E0}{2\,(1+\mu)} \end{pmatrix}$$

```
uVekt = {
  {u1},
  {u2},
  {u3}
 }
```

```
{{u1}, {u2}, {u3}}
```

```
GScher = E0 / 2 / (1 + μ)
```

$$\frac{E0}{2\,(1+\mu)}$$

```
HMatBalken = DiagonalMatrix[{E0, E0, E0, G0, G0, G0}];
HMatBalken // MatrixForm
```

$$\begin{pmatrix} E0 & 0 & 0 & 0 & 0 & 0 \\ 0 & E0 & 0 & 0 & 0 & 0 \\ 0 & 0 & E0 & 0 & 0 & 0 \\ 0 & 0 & 0 & G0 & 0 & 0 \\ 0 & 0 & 0 & 0 & G0 & 0 \\ 0 & 0 & 0 & 0 & 0 & G0 \end{pmatrix}$$

■ Die Elemente der Verzerrungsmatrix und Spannungsmatrix aus L und H

```
ϵMat = LOperator.uVekt; ϵMat // MatrixForm
```

$$\begin{pmatrix} \text{diff1}\, u1 \\ \text{diff2}\, u2 \\ \text{diff3}\, u3 \\ \text{diff2}\, u1 + \text{diff1}\, u2 \\ \text{diff3}\, u2 + \text{diff2}\, u3 \\ \text{diff3}\, u1 + \text{diff1}\, u3 \end{pmatrix}$$

```
σMat = HMat.ϵ; σMat // MatrixForm
```

$$\begin{pmatrix} \frac{E0\,\epsilon 1\,(1-\mu)}{(1-2\,\mu)\,(1+\mu)} + \frac{E0\,\epsilon 2\,\mu}{(1-2\,\mu)\,(1+\mu)} + \frac{E0\,\epsilon 3\,\mu}{(1-2\,\mu)\,(1+\mu)} \\ \frac{E0\,\epsilon 2\,(1-\mu)}{(1-2\,\mu)\,(1+\mu)} + \frac{E0\,\epsilon 1\,\mu}{(1-2\,\mu)\,(1+\mu)} + \frac{E0\,\epsilon 3\,\mu}{(1-2\,\mu)\,(1+\mu)} \\ \frac{E0\,\epsilon 3\,(1-\mu)}{(1-2\,\mu)\,(1+\mu)} + \frac{E0\,\epsilon 1\,\mu}{(1-2\,\mu)\,(1+\mu)} + \frac{E0\,\epsilon 2\,\mu}{(1-2\,\mu)\,(1+\mu)} \\ \frac{E0\,\gamma 12}{2\,(1+\mu)} \\ \frac{E0\,\gamma 23}{2\,(1+\mu)} \\ \frac{E0\,\gamma 31}{2\,(1+\mu)} \end{pmatrix}$$

```
σMat /. μ → 0
```

$$\left\{\{E0\,\epsilon 1\},\ \{E0\,\epsilon 2\},\ \{E0\,\epsilon 3\},\ \left\{\frac{E0\,\gamma 12}{2}\right\},\ \left\{\frac{E0\,\gamma 23}{2}\right\},\ \left\{\frac{E0\,\gamma 31}{2}\right\}\right\}$$

```
σMatBalken = HMatBalken.ϵ /. {ϵ2 → 0, ϵ3 → 0, γ23 → 0};
σMatBalken // MatrixForm
```

$$\begin{pmatrix} E0\,\epsilon 1 \\ 0 \\ 0 \\ G0\,\gamma 12 \\ 0 \\ G0\,\gamma 31 \end{pmatrix}$$