MDA-Chapter 3: Design of Mechanisms - 3.4 Paths of Crank-Rocker Mechanisms

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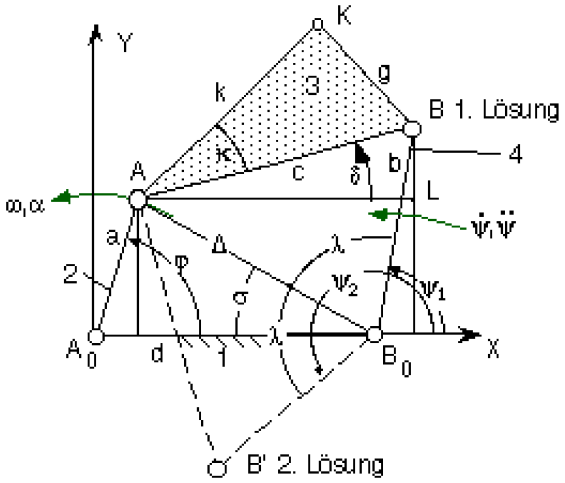
updated 17.11.2009 - Animation

crank-rocker mechanismus: Basic functions




```
Clear[par,φ,ψ,ψ1,ψ2,φ0,ψ0,a,b,c,d];

X = d - a Cos[φ];
Y = a Sin[φ];
σ = ArcTan[X,Y];
Δ = Sqrt[X^2 + Y^2];
λ = Simplify[ArcCos[(Δ^2 + b^2 - c^2) / (2 b Δ)] ];
fψ = π - (σ + λ)
fψ2 = π - (σ - λ)
```

$$\pi - \text{ArcCos}\left[\frac{a^2 + b^2 - c^2 + d^2 - 2\,a\,d\,\text{Cos}[\varphi]}{2\,b\,\sqrt{a^2 + d^2 - 2\,a\,d\,\text{Cos}[\varphi]}}\right] - \text{ArcTan}[d - a\,\text{Cos}[\varphi],\, a\,\text{Sin}[\varphi]]$$

$$\pi + \text{ArcCos}\left[\frac{a^2 + b^2 - c^2 + d^2 - 2\,a\,d\,\text{Cos}[\varphi]}{2\,b\,\sqrt{a^2 + d^2 - 2\,a\,d\,\text{Cos}[\varphi]}}\right] - \text{ArcTan}[d - a\,\text{Cos}[\varphi],\, a\,\text{Sin}[\varphi]]$$

```
φi = Pi + ArcCos[(d^2 + (c - a)^2 - b^2) / (2 (c - a) d)];
φa = ArcCos[(d^2 + (c + a)^2 - b^2) / (2 (c + a) d)];
fφ0 = φi - φa
```

$$\pi + \text{ArcCos}\left[\frac{-b^2 + (-a + c)^2 + d^2}{2\,(-a + c)\,d}\right] - \text{ArcCos}\left[\frac{-b^2 + (a + c)^2 + d^2}{2\,(a + c)\,d}\right]$$

```
ψi = Pi - ArcCos[(d^2 + b^2 - (c - a)^2) / (2 b d)];
ψa = Pi - ArcCos[(d^2 + b^2 - (c + a)^2) / (2 b d)];
fψ0 = ψi - ψa
```

$$-\text{ArcCos}\left[\frac{b^2 - (-a + c)^2 + d^2}{2\,b\,d}\right] + \text{ArcCos}\left[\frac{b^2 - (a + c)^2 + d^2}{2\,b\,d}\right]$$

```
μmin1 = ArcCos[(c^2 + b^2 - (d - a)^2) / (2 b c)]
```

$$\text{ArcCos}\left[\frac{b^2 + c^2 - (-a + d)^2}{2\,b\,c}\right]$$

```
μmin2 = Pi - ArcCos[(c^2 + b^2 - (d + a)^2) / (2 b c)]
```

$$\pi - \text{ArcCos}\left[\frac{b^2 + c^2 - (a + d)^2}{2\,b\,c}\right]$$

```
Grashof = Min[a, b, c, d] + Max[a, b, c, d]
```

```
Max[a, b, c, d] + Min[a, b, c, d]
```

```
fδ = ArcTan[(d + b Cos[ψ] - a Cos[φ]), (b Sin[ψ] - a Sin[φ])]
```

```
ArcTan[d - a Cos[φ] + b Cos[ψ], -a Sin[φ] + b Sin[ψ]]
```

```
xK = A0x + a Cos[φ + γ0] + k Cos[κ + δ + γ0]
yK = A0y + a Sin[φ + γ0] + k Sin[κ + δ + γ0]
```

```
A0x + k Cos[γ0 + δ + κ] + a Cos[γ0 + φ]
```

```
A0y + k Sin[γ0 + δ + κ] + a Sin[γ0 + φ]
```

3.3.1 Find mechanism for path of a coupler point K

```
Clear[par, φ, ψ, ψ1, ψ2, φ0, ψ0, a, b, c, d, k, κ];
```

- **Requirements of crank-rocker mechanism are 5 points of K and par**

```
xK
yK
```

```
A0x + k Cos[γ0 + δ + κ] + a Cos[γ0 + φ]
```

```
A0y + k Sin[γ0 + δ + κ] + a Sin[γ0 + φ]
```

```
par = {a → 26, d → 20, κ -> 0, k → 32}
```

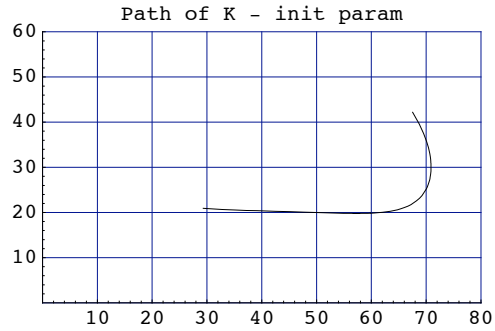
```
{a → 26, d → 20, κ → 0, k → 32}
```

```
R2 = 62  == Simplify[xK /. δ -> fδ /. ψ -> fψ /. par /. φ -> φ1];
R3 = 22  == Simplify[yK /. δ -> fδ /. ψ -> fψ /. par /. φ -> φ1];
R4 = 39  == Simplify[xK /. δ -> fδ /. ψ -> fψ /. par /. φ -> φ2];
R5 = 22  == Simplify[yK /. δ -> fδ /. ψ -> fψ /. par /. φ -> φ2];
R6 = 30  == Simplify[xK /. δ -> fδ /. ψ -> fψ /. par /. φ -> φ3];
R7 = 22  == Simplify[yK /. δ -> fδ /. ψ -> fψ /. par /. φ -> φ3];
R8 = 14  == Simplify[xK /. δ -> fδ /. ψ -> fψ /. par /. φ -> φ4];
R9 = 57  == Simplify[yK /. δ -> fδ /. ψ -> fψ /. par /. φ -> φ4];
R10 = 57  == Simplify[xK /. δ -> fδ /. ψ -> fψ /. par /. φ -> φ5];
R11 = 55  == Simplify[yK /. δ -> fδ /. ψ -> fψ /. par /. φ -> φ5];
```

```
parinit = {A0x → 13, A0y → 33, γ0 → -60 Degree, b → 29, c → 9,
  φ1 → 79 Degree, φ2 → 106 Degree, φ3 → 110 Degree, φ4 → 80 Degree, φ5 → 50 Degree}
```

```
{A0x → 13, A0y → 33, γ0 → -60 °, b → 29, c → 9, φ1 → 79 °, φ2 → 106 °, φ3 → 110 °, φ4 → 80 °, φ5 → 50 °}
```

```
xKpar = xK /. δ → fδ /. ψ → fψ /. par /. parinit;
yKpar = yK /. δ → fδ /. ψ → fψ /. par /. parinit;
ParametricPlot[{xKpar, yKpar}, {φ, 50 °, 110 °}, PlotRange → {{0, 80}, {0, 60}},
 GridLines → Automatic, PlotLabel → "Path of K - init param"]
```




```
erg1 = FindRoot[{R2, R3, R4, R5, R6, R7, R8, R9, R10, R11},
  {{A0x, 13}, {A0y, 33}, {γ0, -60 Degree}, {b, 29}, {c, 9}, {φ1, 79 Degree},
   {φ2, 106 Degree}, {φ3, 110 Degree}, {φ4, 80 Degree}, {φ5, 50 Degree}}]
```

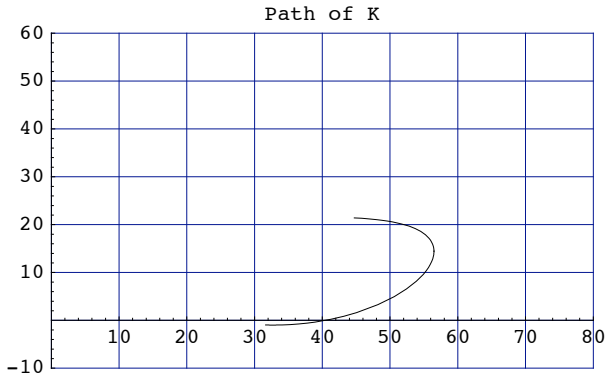
```
FindRoot::lstol :
 The line search decreased the step size to within tolerance specified
   by AccuracyGoal and PrecisionGoal but was unable to find a sufficient
   decrease in the merit function.  You may need more than MachinePrecision
   digits of working precision to meet these tolerances. Mehr…
```

```
{A0x → 13.6795, A0y → 37.9567, γ0 → -0.299599, b → 11.9464, c → 9.84789,
 φ1 → 0.499533, φ2 → 0.888403, φ3 → 0.898244, φ4 → 0.526628, φ5 → -0.0186611}
```

```
par
```

```
{a → 26, d → 20, κ → 0, k → 32}
```

```
xKpar = xK /. δ → fδ /. ψ → fψ /. erg1 /. par;
yKpar = yK /. δ → fδ /. ψ → fψ /. erg1 /. par;
ParametricPlot[{xKpar, yKpar}, {φ, 0 °, 50 °},
 PlotRange → {{0, 80}, {-10, 60}}, GridLines → Automatic, PlotLabel → "Path of K"]
```



No realistic solution is found

■ Now we use the optimisation method

```
xCGi = {62, 39, 30, 14, 57}
yCGi = {22, 22, 22, 57, 55}
```

```
{62, 39, 30, 14, 57}
```

```
{22, 22, 22, 57, 55}
```

```
φi = {φ1, φ2, φ3, φ4, φ5}
```

```
{φ1, φ2, φ3, φ4, φ5}
```

```
Simplify[xK /. δ -> fδ /. ψ -> fψ /. φ -> φi[[1]]] - xCGi[[1]]
```

$$-62 + A0x + a\,\text{Cos}[\gamma 0 + \varphi 1] + k\,\text{Cos}\left[\gamma 0 + \varkappa + \text{ArcTan}\left[\frac{-a\,\text{Sin}[\varphi 1] + b\,\text{Sin}\left[\text{ArcCos}\left[\frac{a^2+b^2-c^2+d^2-2\,a\,d\,\text{Cos}[\varphi 1]}{2\,b\,\sqrt{a^2+d^2-2\,a\,d\,\text{Cos}[\varphi 1]}}\right] + \text{ArcTan}[d - a\,\text{Cos}[\varphi 1],\, a\,\text{Sin}[\varphi 1]]\right]}{d - a\,\text{Cos}[\varphi 1] - b\,\text{Cos}\left[\text{ArcCos}\left[\frac{a^2+b^2-c^2+d^2-2\,a\,d\,\text{Cos}[\varphi 1]}{2\,b\,\sqrt{a^2+d^2-2\,a\,d\,\text{Cos}[\varphi 1]}}\right] + \text{ArcTan}[d - a\,\text{Cos}[\varphi 1],\, a\,\text{Sin}[\varphi 1]]\right]}\right]\right]$$

```
Simplify[yK /. δ -> fδ /. ψ -> fψ /. φ -> φi[[1]]] - yCGi[[1]]
```

$$-22 + A0y + a\,\text{Sin}[\gamma 0 + \varphi 1] + k\,\text{Sin}\left[\gamma 0 + \varkappa + \text{ArcTan}\left[\frac{-a\,\text{Sin}[\varphi 1] + b\,\text{Sin}\left[\text{ArcCos}\left[\frac{a^2+b^2-c^2+d^2-2\,a\,d\,\text{Cos}[\varphi 1]}{2\,b\,\sqrt{a^2+d^2-2\,a\,d\,\text{Cos}[\varphi 1]}}\right] + \text{ArcTan}[d - a\,\text{Cos}[\varphi 1],\, a\,\text{Sin}[\varphi 1]]\right]}{d - a\,\text{Cos}[\varphi 1] - b\,\text{Cos}\left[\text{ArcCos}\left[\frac{a^2+b^2-c^2+d^2-2\,a\,d\,\text{Cos}[\varphi 1]}{2\,b\,\sqrt{a^2+d^2-2\,a\,d\,\text{Cos}[\varphi 1]}}\right] + \text{ArcTan}[d - a\,\text{Cos}[\varphi 1],\, a\,\text{Sin}[\varphi 1]]\right]}\right]\right]$$

we have 14 parameters:

```
par
```

```
{a → 26, d → 20, ϰ → 0, k → 32}
```

```
parinit
```

```
{A0x → 13, A0y → 33, γ0 → -60 °, b → 29, c → 9, φ1 → 79 °, φ2 → 106 °, φ3 → 110 °, φ4 → 80 °, φ5 → 50 °}
```

```
ferror = Sum[(Simplify[xK /. δ -> fδ /. ψ -> fψ /. φ -> φi[[i]]] - xCGi[[i]]) ^ 2 +
            (Simplify[yK /. δ -> fδ /. ψ -> fψ /. φ -> φi[[i]]] - yCGi[[i]]) ^ 2,
    {i, 1, 3}] // N;
```

```
erg2 = NMinimize[{ferror, 10 ≤ A0x ≤ 15, 30 ≤ A0y ≤ 35, -70 Degree ≤ γ0 ≤ -50 Degree, 25 ≤ a ≤ 30,
   25 ≤ b ≤ 35, 5 ≤ c ≤ 12, 18 ≤ d ≤ 25, 30 ≤ k ≤ 50, -0.1 ≤ ϰ ≤ 0.1, 0 ≤ φ1 ≤ 2, 0 ≤ φ2 ≤ 2,
   0 ≤ φ3 ≤ 2, 0 ≤ φ4 ≤ 2, 0 ≤ φ5 ≤ 2}, {A0x, A0y, γ0, a, b, c, d, k, ϰ, φ1, φ2, φ3, φ4, φ5}]
```

```
NMinimize::nrnum : The function value 9388.75 + 15394.5 i
   is not a real number at {a, A0x, A0y, b, c, d, k, γ0, ϰ, φ1, <<4>>} =
   {25.1593, 13.201, 33.6215, 30.8088, 11.5181, <<19>>, <<18>>, -1.20465, 0.0404607,
    0.850468, <<4>>}. Mehr…
```

Also this solution fail. reason is that the proposed path is not possible with this desired lengths of bars !