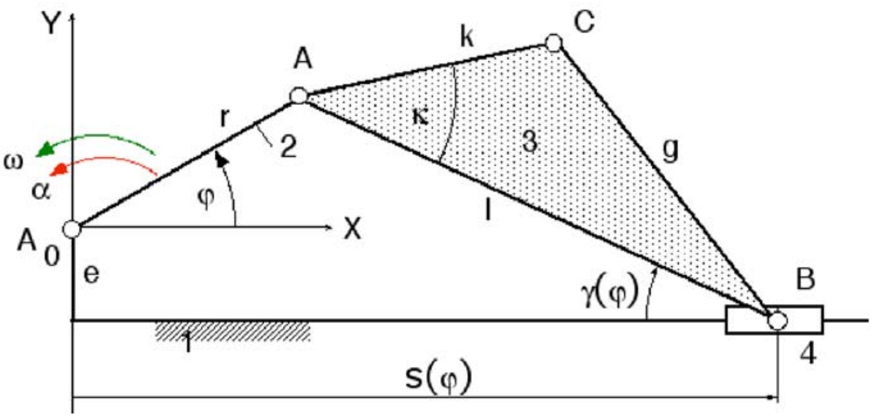
# Problem 4.04: Kinematics of a Slider Crank Mechanism

22.05.2008




## 1. Transfer functions s(phi), s'= s1, s" = s2

```
Clear[phi,omega,alpha,gamma,dgamma,ddgamma,r,l,e,k,kappa]
```

```
rax = r Cos[phi]
```

$r\,\mathrm{Cos}[phi]$

```
ray = r Sin[phi]
```

$r\,\mathrm{Sin}[phi]$

```
s = rax + Sqrt[l^2 - (ray + e)^2]
```

$$r\,\mathrm{Cos}[phi] + \sqrt{l^2 - (e + r\,\mathrm{Sin}[phi])^2}$$

```
s1 = D[s, phi]
```

$$-r\,\mathrm{Sin}[phi] - \frac{r\,\mathrm{Cos}[phi]\,(e + r\,\mathrm{Sin}[phi])}{\sqrt{l^2 - (e + r\,\mathrm{Sin}[phi])^2}}$$

```
s2 = D[s1, phi]
```

$$-r\,\mathrm{Cos}[phi] - \frac{r^2\,\mathrm{Cos}[phi]^2\,(e + r\,\mathrm{Sin}[phi])^2}{\left(l^2 - (e + r\,\mathrm{Sin}[phi])^2\right)^{3/2}} - \frac{r^2\,\mathrm{Cos}[phi]^2}{\sqrt{l^2 - (e + r\,\mathrm{Sin}[phi])^2}} + \frac{r\,\mathrm{Sin}[phi]\,(e + r\,\mathrm{Sin}[phi])}{\sqrt{l^2 - (e + r\,\mathrm{Sin}[phi])^2}}$$

## 2. Velocity vB= ds/dt and Acceleration aB = dvB/dt of point B

```
vB = s1 ω
```

$$\omega\left(-r\,\mathrm{Sin}[phi] - \frac{r\,\mathrm{Cos}[phi]\,(e + r\,\mathrm{Sin}[phi])}{\sqrt{l^2 - (e + r\,\mathrm{Sin}[phi])^2}}\right)$$

```
aB = s1 α + s2 ω^2
```

$$\alpha\left(-r\,\mathrm{Sin}[phi] - \frac{r\,\mathrm{Cos}[phi]\,(e + r\,\mathrm{Sin}[phi])}{\sqrt{l^2 - (e + r\,\mathrm{Sin}[phi])^2}}\right) + \omega^2$$
$$\left(-r\,\mathrm{Cos}[phi] - \frac{r^2\,\mathrm{Cos}[phi]^2\,(e + r\,\mathrm{Sin}[phi])^2}{\left(l^2 - (e + r\,\mathrm{Sin}[phi])^2\right)^{3/2}} - \frac{r^2\,\mathrm{Cos}[phi]^2}{\sqrt{l^2 - (e + r\,\mathrm{Sin}[phi])^2}} + \frac{r\,\mathrm{Sin}[phi]\,(e + r\,\mathrm{Sin}[phi])}{\sqrt{l^2 - (e + r\,\mathrm{Sin}[phi])^2}}\right)$$

## 3. Coupling point C:

position rcx und rcy (phi, gamma)
velocity vcx, vcy (phi, omega, gamma, dgamma)
acceleration acx, acy (phi, omega, alpha, gamma, dgamma, ddgamma)

```
rbx = s; rby = -e;
```

```
Clear [gamma,dgamma,ddgamma,rcx, rcy, vcx, vcy, acx, acy]
rcx = rax + k Cos[kappa - gamma]
```

$$k\,\mathrm{Cos}[\mathrm{gamma}-\mathrm{kappa}]+r\,\mathrm{Cos}[\mathrm{phi}]$$

```
rcy = ray + k Sin[kappa - gamma]
```

$$-k\,\mathrm{Sin}[\mathrm{gamma}-\mathrm{kappa}]+r\,\mathrm{Sin}[\mathrm{phi}]$$

```
vcx = D[rcx, t, NonConstants->{phi,gamma}]/.
                     {D[phi,  t, NonConstants->{phi,gamma}]->ω,
                      D[gamma,t, NonConstants->{phi,gamma}]->dgamma}
```

$$-\mathrm{dgamma}\,k\,\mathrm{Sin}[\mathrm{gamma}-\mathrm{kappa}]-r\,\omega\,\mathrm{Sin}[\mathrm{phi}]$$

```
vcy = D[rcy, t, NonConstants->{phi,gamma}]/.
                     {D[phi,  t, NonConstants->{phi,gamma}]->ω,
                      D[gamma,t, NonConstants->{phi,gamma}]->dgamma}
```

$$-\mathrm{dgamma}\,k\,\mathrm{Cos}[\mathrm{gamma}-\mathrm{kappa}]+r\,\omega\,\mathrm{Cos}[\mathrm{phi}]$$

```
acx = D[rcx, {t,2}, NonConstants->{phi,gamma}]/.
                           {D[phi,t,NonConstants->{phi,gamma}]->ω,
                            D[phi,  {t,2},NonConstants->{phi,gamma}]->α,
                            D[gamma , t,  NonConstants->{phi,gamma}]->dgamma,
                            D[gamma,{t,2},NonConstants->{phi,gamma}]->ddgamma}
```

$$k\left(-\mathrm{dgamma}^2\,\mathrm{Cos}[\mathrm{gamma}-\mathrm{kappa}]-\mathrm{ddgamma}\,\mathrm{Sin}[\mathrm{gamma}-\mathrm{kappa}]\right)+r\left(-\omega^2\,\mathrm{Cos}[\mathrm{phi}]-\alpha\,\mathrm{Sin}[\mathrm{phi}]\right)$$

```
acy = D[rcy, {t,2}, NonConstants->{phi,gamma}]/.
                           {D[phi,t,NonConstants->{phi,gamma}]->ω,
                            D[phi,  {t,2},NonConstants->{phi,gamma}]->α,
                            D[gamma , t,  NonConstants->{phi,gamma}]->dgamma,
                            D[gamma,{t,2},NonConstants->{phi,gamma}]->ddgamma}
```

$$-k\left(\mathrm{ddgamma}\,\mathrm{Cos}[\mathrm{gamma}-\mathrm{kappa}]-\mathrm{dgamma}^2\,\mathrm{Sin}[\mathrm{gamma}-\mathrm{kappa}]\right)+r\left(\alpha\,\mathrm{Cos}[\mathrm{phi}]-\omega^2\,\mathrm{Sin}[\mathrm{phi}]\right)$$

```
vC = Simplify [Sqrt[vcx^2 + vcy^2] ]
```

$$\sqrt{\mathrm{dgamma}^2\,k^2+r^2\,\omega^2-2\,\mathrm{dgamma}\,k\,r\,\omega\,\mathrm{Cos}[\mathrm{gamma}-\mathrm{kappa}+\mathrm{phi}]}$$

```
aC = Simplify [Sqrt[acx^2 + acy^2] ]
```

$$\sqrt{\left(\left(\mathrm{dgamma}^2\,k\,\mathrm{Cos}[\mathrm{gamma}-\mathrm{kappa}]+r\,\omega^2\,\mathrm{Cos}[\mathrm{phi}]+\mathrm{ddgamma}\,k\,\mathrm{Sin}[\mathrm{gamma}-\mathrm{kappa}]+r\,\alpha\,\mathrm{Sin}[\mathrm{phi}]\right)^2+\left(\mathrm{ddgamma}\,k\,\mathrm{Cos}[\mathrm{gamma}-\mathrm{kappa}]-r\,\alpha\,\mathrm{Cos}[\mathrm{phi}]-\mathrm{dgamma}^2\,k\,\mathrm{Sin}[\mathrm{gamma}-\mathrm{kappa}]+r\,\omega^2\,\mathrm{Sin}[\mathrm{phi}]\right)^2\right)}$$

### ■ auxiliary angle gamma

```
gamma = ArcSin[(ray + e) / l]
```

$$\mathrm{ArcSin}\left[\frac{e+r\,\mathrm{Sin}[\mathrm{phi}]}{l}\right]$$

```
dgamma = Simplify[D[gamma, t, NonConstants → {phi}] /. {D[phi, t, NonConstants → {phi}] → ω}]
```

$$\frac{r\,\omega\,\mathrm{Cos}[\mathrm{phi}]}{l\sqrt{1-\frac{(e+r\,\mathrm{Sin}[\mathrm{phi}])^2}{l^2}}}$$

```
ddgamma = D[gamma, {t, 2}, NonConstants → {phi}] /.
  {D[phi, t, NonConstants → {phi}] → ω, D[phi, {t, 2}, NonConstants → {phi}] → α}
```

$$\frac{r^2\,\omega^2\,\mathrm{Cos}[\mathrm{phi}]^2\,(e+r\,\mathrm{Sin}[\mathrm{phi}])}{l^3\left(1-\frac{(e+r\,\mathrm{Sin}[\mathrm{phi}])^2}{l^2}\right)^{3/2}}+\frac{r\,\alpha\,\mathrm{Cos}[\mathrm{phi}]}{l\sqrt{1-\frac{(e+r\,\mathrm{Sin}[\mathrm{phi}])^2}{l^2}}}-\frac{r\,\omega^2\,\mathrm{Sin}[\mathrm{phi}]}{l\sqrt{1-\frac{(e+r\,\mathrm{Sin}[\mathrm{phi}])^2}{l^2}}}$$

```
vcx
```

$$-r\,\omega\,\mathrm{Sin}[\mathrm{phi}]+\frac{k\,r\,\omega\,\mathrm{Cos}[\mathrm{phi}]\,\mathrm{Sin}\left[\mathrm{kappa}-\mathrm{ArcSin}\left[\frac{e+r\,\mathrm{Sin}[\mathrm{phi}]}{l}\right]\right]}{l\sqrt{1-\frac{(e+r\,\mathrm{Sin}[\mathrm{phi}])^2}{l^2}}}$$

```
acx
```

$$r\left(-\omega^2\,\mathrm{Cos}[\mathrm{phi}]-\alpha\,\mathrm{Sin}[\mathrm{phi}]\right)+k\left(-\frac{r^2\,\omega^2\,\mathrm{Cos}[\mathrm{phi}]^2\,\mathrm{Cos}\left[\mathrm{kappa}-\mathrm{ArcSin}\left[\frac{e+r\,\mathrm{Sin}[\mathrm{phi}]}{l}\right]\right]}{l^2\left(1-\frac{(e+r\,\mathrm{Sin}[\mathrm{phi}])^2}{l^2}\right)}+\left(\frac{r^2\,\omega^2\,\mathrm{Cos}[\mathrm{phi}]^2\,(e+r\,\mathrm{Sin}[\mathrm{phi}])}{l^3\left(1-\frac{(e+r\,\mathrm{Sin}[\mathrm{phi}])^2}{l^2}\right)^{3/2}}+\frac{r\,\alpha\,\mathrm{Cos}[\mathrm{phi}]}{l\sqrt{1-\frac{(e+r\,\mathrm{Sin}[\mathrm{phi}])^2}{l^2}}}-\frac{r\,\omega^2\,\mathrm{Sin}[\mathrm{phi}]}{l\sqrt{1-\frac{(e+r\,\mathrm{Sin}[\mathrm{phi}])^2}{l^2}}}\right)\mathrm{Sin}\left[\mathrm{kappa}-\mathrm{ArcSin}\left[\frac{e+r\,\mathrm{Sin}[\mathrm{phi}]}{l}\right]\right]\right)$$

## ■ 4.a evaluation of functions for given values phi, $\omega$, $\alpha$

Alle Längen in cm, Winkel in Grad.

```
par1 = {r -> 30, l -> 60, e -> 10, k -> 30, kappa -> 38 Degree, α → 20}
```

```
{r → 30, l → 60, e → 10, k → 30, kappa → 38 °, α → 20}
```

```
par2 = {phi -> 30 Degree, ω -> 4, α ->20}
```

```
{phi → 30 °, ω → 4, α → 20}
```

```
{s, s1, s2 }/.par1/.par2 //N
```

```
{80.5243, -26.9083, -34.0808}
```

```
{vB, aB}/.par1/.par2  //N
```

```
{-107.633, -1083.46}
```

```
{rcx, rcy, vcx, vcy, vC, acx, acy, aC}/.par1/.par2 //N
```

```
{55.167, 21.9401, -46.777, 48.3139, 67.2482, -774.52, 56.2355, 776.559}
```

## ■ 5. Plots of functions for $\alpha$ = 20 rad/s = const.

$\omega$ = int($\alpha$ dt) = $\alpha$ t + $\omega$0,   phi = int($\alpha$ dt) = 1/2 $\alpha$ t^2 + $\omega$0 t + phi0:  $\omega$0 = phi0 = 0:

```
ωt = α t /. par2
```

```
20 t
```

```
phit = 1 / 2 α t ^ 2 /. par2
```

$10\ t^2$

```
tt = Sqrt[4 Pi / 20] // N
```

0.792665

```
phiG = phi / Degree /. phi -> phit;
gammaG = gamma / Degree /. par1 /. phi -> phit
sp = s /. par1 /. phi -> phit
vBp = vB /. par1 /. phi -> phit /. ω → ωt
aBp = aB /. par1 /. phi -> phit /. ω → ωt
```

$$\frac{\text{ArcSin}\left[\frac{1}{60}\left(10+30\,\text{Sin}\left[10\,t^2\right]\right)\right]}{\circ}$$

$$30\,\text{Cos}\left[10\,t^2\right]+\sqrt{3600-\left(10+30\,\text{Sin}\left[10\,t^2\right]\right)^2}$$

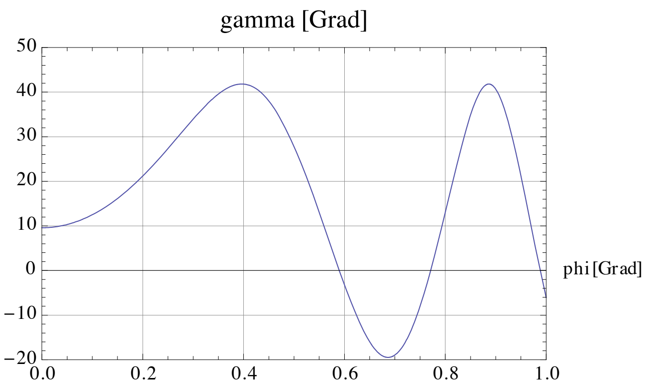
$$20\,t\left(-30\,\text{Sin}\left[10\,t^2\right]-\frac{30\,\text{Cos}\left[10\,t^2\right]\left(10+30\,\text{Sin}\left[10\,t^2\right]\right)}{\sqrt{3600-\left(10+30\,\text{Sin}\left[10\,t^2\right]\right)^2}}\right)$$

$$20\left(-30\,\text{Sin}\left[10\,t^2\right]-\frac{30\,\text{Cos}\left[10\,t^2\right]\left(10+30\,\text{Sin}\left[10\,t^2\right]\right)}{\sqrt{3600-\left(10+30\,\text{Sin}\left[10\,t^2\right]\right)^2}}\right)+$$

$$400\,t^2\left(-30\,\text{Cos}\left[10\,t^2\right]-\frac{900\,\text{Cos}\left[10\,t^2\right]^2\left(10+30\,\text{Sin}\left[10\,t^2\right]\right)^2}{\left(3600-\left(10+30\,\text{Sin}\left[10\,t^2\right]\right)^2\right)^{3/2}}-\right.$$

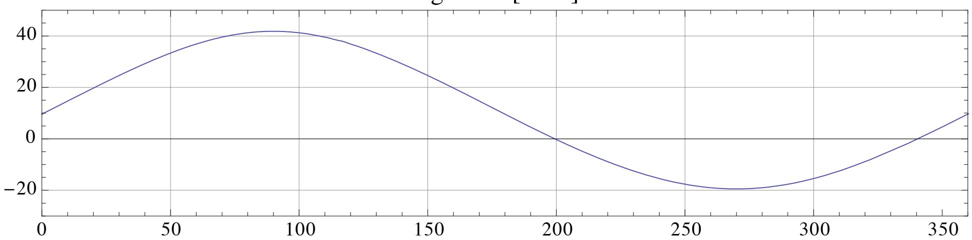
$$\left.\frac{900\,\text{Cos}\left[10\,t^2\right]^2}{\sqrt{3600-\left(10+30\,\text{Sin}\left[10\,t^2\right]\right)^2}}+\frac{30\,\text{Sin}\left[10\,t^2\right]\left(10+30\,\text{Sin}\left[10\,t^2\right]\right)}{\sqrt{3600-\left(10+30\,\text{Sin}\left[10\,t^2\right]\right)^2}}\right)$$

```
Plot[gammaG, {t, 0, 1}, Frame → True, GridLines → Automatic,
 AxesLabel → {"phi  [Grad]", None}, PlotLabel → "gamma  [Grad]", PlotRange → {{0, 1}, {-20, 50}}]
```

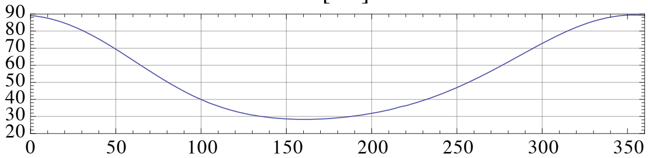



```
phiG = phi / Degree /. phi -> phit;
```

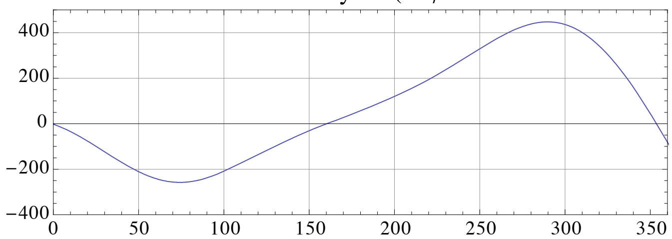
```
ParametricPlot[{phiG, gammaG}, {t, 0, 0.8}, Frame → True, GridLines → Automatic,
 AxesLabel → {"phi [Grad]", None}, PlotLabel → "gamma [Grad]", PlotRange → {{0, 360}, {-30, 50}}]
```



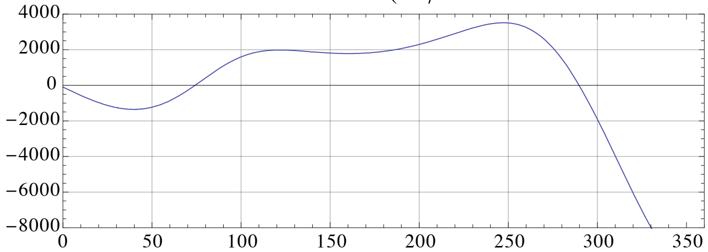

```
ParametricPlot[{phiG, sp}, {t, 0, 0.8}, Frame → True, GridLines → Automatic,
 AxesLabel → {"phi [Grad]", None}, PlotLabel → "s [cm]", PlotRange → {{0, 360}, {20, 90}}]
```




```
ParametricPlot[{phiG, vBp}, {t, 0, 0.8}, Frame → True, GridLines → Automatic,
 AxesLabel → {"phi [Grad]", None}, PlotLabel → "velocity vB (cm/s",
 PlotRange → {{0, 360}, {-400, 500}}, AspectRatio → 1 / 3]
```




```
ParametricPlot[{phiG, aBp}, {t, 0, 0.8}, Frame → True, GridLines → Automatic,
 AxesLabel → {"phi [Grad]", None}, PlotLabel → "accel aB (cm/s^2",
 PlotRange → {{0, 360}, {-8000, 4000}}, AspectRatio → 1 / 3]
```



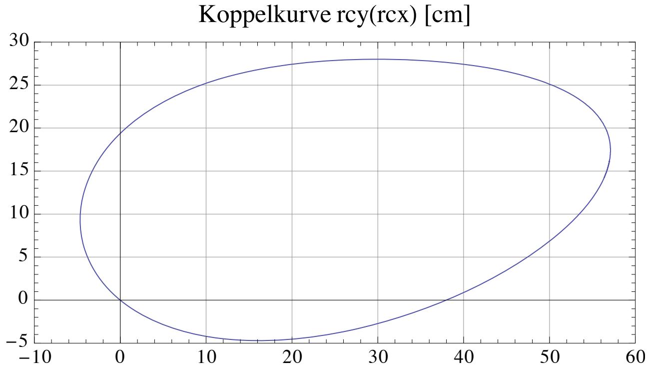

```
rcxp = rcx /. par1 /. phi -> phit /. ω → ωt
```

$$30\,\mathrm{Cos}\left[10\,t^2\right] + 30\,\mathrm{Cos}\left[38\,° - \mathrm{ArcSin}\left[\frac{1}{60}\left(10 + 30\,\mathrm{Sin}\left[10\,t^2\right]\right)\right]\right]$$

```
rcyp = rcy /. par1 /. phi -> phit /. ω → ωt
```

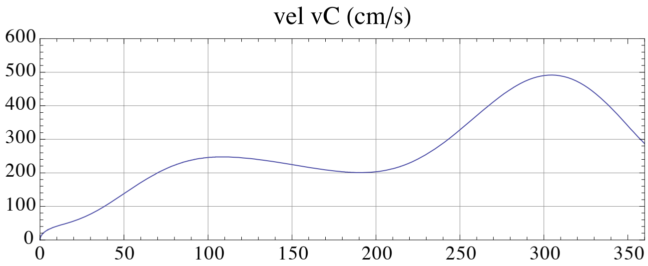
$$30\,\mathrm{Sin}\left[10\,t^2\right] + 30\,\mathrm{Sin}\left[38\,° - \mathrm{ArcSin}\left[\frac{1}{60}\left(10 + 30\,\mathrm{Sin}\left[10\,t^2\right]\right)\right]\right]$$

```
Plokurve = ParametricPlot[{rcxp, rcyp}, {t, 0, 0.8},
  Frame → True, GridLines → Automatic, PlotLabel → "Koppelkurve rcy(rcx)  [cm]",
  AxesLabel → {None, None}, PlotRange → {{-10, 60}, {-5, 30}}]
```




```
vCp = vC /. par1 /. phi -> phit /. ω → ωt;
aCp = aC /. par1 /. phi -> phit /. ω → ωt;
```

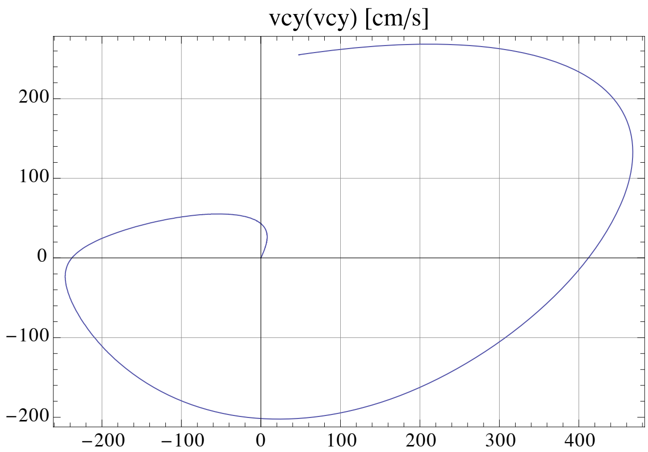
```
ParametricPlot[{phiG, vCp}, {t, 0, 0.8}, Frame → True,
 GridLines → Automatic, AxesLabel → {"phi  [Grad]", None},
 PlotLabel → "vel vC  (cm/s)", PlotRange → {{0, 360}, {-0, 600}}, AspectRatio → 1 / 3]
```



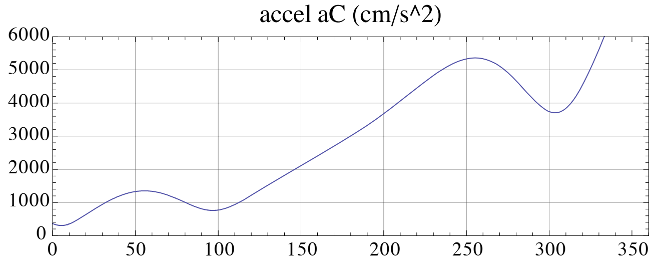

```
vcx /. par1 /. phi -> phit /. ω → ωt
```

$$-600\,t\,\mathrm{Sin}\left[10\,t^2\right] + \frac{300\,t\,\mathrm{Cos}\left[10\,t^2\right]\,\mathrm{Sin}\left[38\,° - \mathrm{ArcSin}\left[\frac{1}{60}\,\left(10 + 30\,\mathrm{Sin}\left[10\,t^2\right]\right)\right]\right]}{\sqrt{1 - \frac{\left(10+30\,\mathrm{Sin}\left[10\,t^2\right]\right)^2}{3600}}}$$

```
ParametricPlot[{vcx /. par1 /. phi -> phit /. ω → ωt, vcy /. par1 /. phi -> phit /. ω → ωt},
 {t, 0, 0.8}, Frame → True, GridLines → Automatic, PlotLabel → "vcy(vcy)  [cm/s]",
 AxesLabel → {None, None}, PlotRange → {Automatic, Automatic}]
```

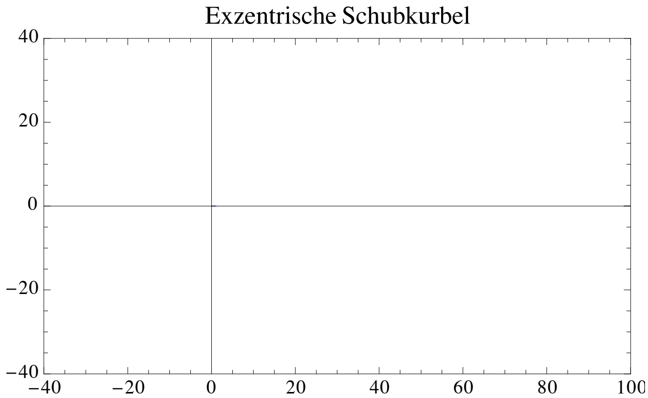


```
ParametricPlot[{phiG, aCp}, {t, 0, 0.8}, Frame → True, GridLines → Automatic,
 AxesLabel → {"phi [Grad]", None}, PlotLabel → "accel aC (cm/s^2)",
 PlotRange → {{0, 360}, {-0, 6000}}, AspectRatio → 1 / 3]
```

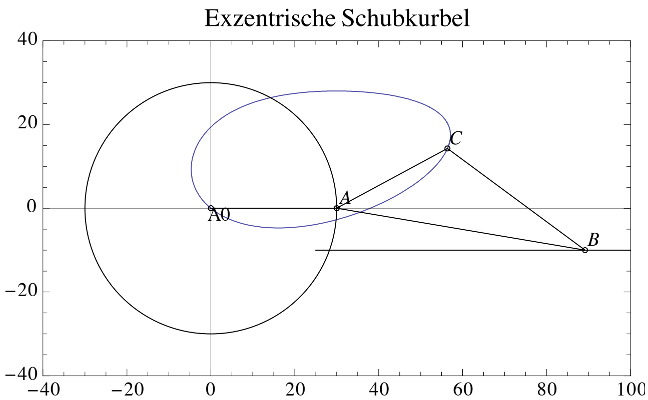



## Animation der Schubkurbel

```
Clear[phi]
lmax = Max[r, l, k] /. par1; r = r /. par1; l = l /. par1;
e = e /. par1; k = k /. par1; kappa = kappa /. par1;
Plo0 = Plot[0, {i, 0, 1}, PlotRange → {{-40, 100}, {-40, 40}},
  Frame → True, AspectRatio → Automatic, PlotLabel → "Exzentrische Schubkurbel"]
```




```
Do[PloMecha1 = Graphics[{Circle[{0, 0}, lmax/100], Circle[{0, 0}, r], Circle[{rax, ray}, lmax/100],
    Circle[{rbx, rby}, lmax/100], Circle[{rcx, rcy}, lmax/100], Line[{{0, 0}, {rax, ray}}],
    Line[{{25, rby}, {100, rby}}], Line[{{rax, ray}, {rbx, rby}, {rcx, rcy}, {rax, ray}}],
    Text[A0, {lmax/30, -lmax/30}], Text[A, {rax + lmax/30, ray + lmax/30}],
    Text[B, {rbx + lmax/30, rby + lmax/30}], Text[C, {rcx + lmax/30, rcy + lmax/30}]}];
  Print[Show[Plo0, Plokurve, PloMecha1]], {phi, 0, 2 π, π/6}];
```




## 7. Input torque M2 = -s´(phi) F0 cos phi and input power P