MDA-Project 1 - Problem 2:
Find a Crank-Rocker with Points C and D for Given Coupler Positions

Part 1: Graphical solution, plot of the mechanism.

▪ crank-rocker mechanismus: Basic functions

```
Clear[par,φ,ψ,ψ1,ψ2,φ0,ψ0,a,b,c,d];
```

```
σ = ArcTan[a Sin[φ]/(d-a Cos[φ])]
Δ = Simplify[Sqrt[(a Sin[φ])^2 + (d-a Cos[φ])^2]]
λ = Simplify[ArcCos[(Δ^2 + b^2 - c^2) / (2 b Δ)] ]
fψ = π - (σ + λ)
fψ2 = π - (σ - λ)
```

$$\mathrm{ArcTan}\left[\frac{a\,\mathrm{Sin}[\varphi]}{d - a\,\mathrm{Cos}[\varphi]}\right]$$

$$\sqrt{a^2 + d^2 - 2\,a\,d\,\mathrm{Cos}[\varphi]}$$

$$\mathrm{ArcCos}\left[\frac{a^2 + b^2 - c^2 + d^2 - 2\,a\,d\,\mathrm{Cos}[\varphi]}{2\,b\,\sqrt{a^2 + d^2 - 2\,a\,d\,\mathrm{Cos}[\varphi]}}\right]$$

$$\pi - \mathrm{ArcCos}\left[\frac{a^2 + b^2 - c^2 + d^2 - 2\,a\,d\,\mathrm{Cos}[\varphi]}{2\,b\,\sqrt{a^2 + d^2 - 2\,a\,d\,\mathrm{Cos}[\varphi]}}\right] - \mathrm{ArcTan}\left[\frac{a\,\mathrm{Sin}[\varphi]}{d - a\,\mathrm{Cos}[\varphi]}\right]$$

$$\pi + \mathrm{ArcCos}\left[\frac{a^2 + b^2 - c^2 + d^2 - 2\,a\,d\,\mathrm{Cos}[\varphi]}{2\,b\,\sqrt{a^2 + d^2 - 2\,a\,d\,\mathrm{Cos}[\varphi]}}\right] - \mathrm{ArcTan}\left[\frac{a\,\mathrm{Sin}[\varphi]}{d - a\,\mathrm{Cos}[\varphi]}\right]$$

```
φi = Pi + ArcCos[(d^2 + (c - a)^2 - b^2) / (2 (c - a) d)];
φa = ArcCos[(d^2 + (c + a)^2 - b^2) / (2 (c + a) d)];
fφ0 = φi - φa
```

$$\pi + \mathrm{ArcCos}\left[\frac{-b^2 + (-a + c)^2 + d^2}{2\,(-a + c)\,d}\right] - \mathrm{ArcCos}\left[\frac{-b^2 + (a + c)^2 + d^2}{2\,(a + c)\,d}\right]$$

```
ψi = Pi - ArcCos[(d^2 + b^2 - (c - a)^2) / (2 b d)];
ψa = Pi - ArcCos[(d^2 + b^2 - (c + a)^2) / (2 b d)];
fψ0 = ψi - ψa
```

$$-\mathrm{ArcCos}\left[\frac{b^2 - (-a + c)^2 + d^2}{2\,b\,d}\right] + \mathrm{ArcCos}\left[\frac{b^2 - (a + c)^2 + d^2}{2\,b\,d}\right]$$

```
μmin1 = ArcCos[(c^2 + b^2 - (d - a)^2) / (2 b c)]
```

$$\mathrm{ArcCos}\left[\frac{b^2 + c^2 - (-a + d)^2}{2\,b\,c}\right]$$

```
μmin2 = Pi - ArcCos[(c^2 + b^2 - (d + a)^2) / (2 b c)]
```

$$\pi - \mathrm{ArcCos}\left[\frac{b^2 + c^2 - (a + d)^2}{2\,b\,c}\right]$$


```
Grashof = Min[a, b, c, d] + Max[a, b, c, d]
```

```
Max[a, b, c, d] + Min[a, b, c, d]
```

```
fδ = ArcTan[(d + b Cos[ψ] - a Cos[φ]), (b Sin[ψ] - a Sin[φ])]
```

```
ArcTan[d - a Cos[φ] + b Cos[ψ], -a Sin[φ] + b Sin[ψ]]
```

point B over B0

```
xB = A0x + b Cos[ψ + γ0] + d Cos[γ0]
yB = A0y + b Sin[ψ + γ0] + d Sin[γ0]
```

```
A0x + d Cos[γ0] + b Cos[γ0 + ψ]

A0y + d Sin[γ0] + b Sin[γ0 + ψ]
```

point C over A

```
xC = A0x + a Cos[φ + γ0] + kC Cos[ϰC + δ + γ0]
yC = A0y + a Sin[φ + γ0] + kC Sin[ϰC + δ + γ0]
```

```
A0x + kC Cos[γ0 + δ + ϰC] + a Cos[γ0 + φ]

A0y + kC Sin[γ0 + δ + ϰC] + a Sin[γ0 + φ]
```

point D over A

```
xD = A0x + a Cos[φ + γ0] + kD Cos[ϰD + δ + γ0]
yD = A0y + a Sin[φ + γ0] + kD Sin[ϰD + δ + γ0]
```

```
A0x + kD Cos[γ0 + δ + ϰD] + a Cos[γ0 + φ]

A0y + kD Sin[γ0 + δ + ϰD] + a Sin[γ0 + φ]
```

▪ Test of functions by a par set 1: graphical solution - non-scaled

Hand solution: par1

```
par1 = {A0x → 0, A0y → 0, γ0 → 0 °, a → 22.3, b → 75, c → 80,
  d → 40.5, kC → 67.5, ϰC → 80 Degree, kD → 72.2, ϰD → 27.5 Degree,
  φ1 → 1.08, φ2 → 180 Degree, φ3 → 2 Pi - 1.0, φ4 → 360 Degree }
```

```
{A0x → 0, A0y → 0, γ0 → 0, a → 22.3, b → 75, c → 80, d → 40.5, kC → 67.5, ϰC → 80 °,
 kD → 72.2, ϰD → 0.479966, φ1 → 1.08, φ2 → 180 °, φ3 → 5.28319, φ4 → 360 °}
```

```
xx = ArcCos[(a^2 + a^2 - 21.5^2) / (2 a a)] /. par1
```

```
1.00602
```

```
(2 Pi - 1.0) / Degree
```

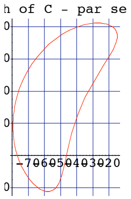
```
302.704
```

```
1.08 / Degree
```

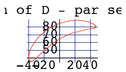
```
61.8794
```

▪ plot of mechanism

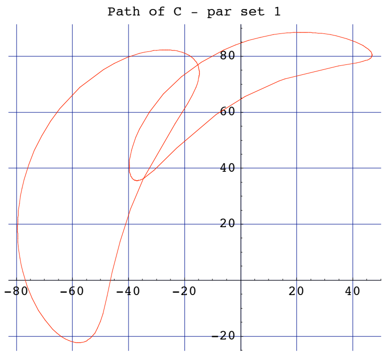
```
xCpar = xC /. δ -> fδ /. ψ -> fψ /. par1;
yCpar = yC /. δ -> fδ /. ψ -> fψ /. par1;
plo1C = ParametricPlot[{xCpar, yCpar}, {φ, 0, 2 Pi},
   GridLines → Automatic, PlotLabel → "Path of C - par set 1",
   AspectRatio → Automatic, PlotStyle → {{RGBColor[1, 0, 0]}}];
```



```
xDpar = xD /. δ -> fδ /. ψ -> fψ /. par1;
yDpar = yD /. δ -> fδ /. ψ -> fψ /. par1;
plo1D = ParametricPlot[{xDpar, yDpar}, {φ, 0, 2 Pi},
   GridLines → Automatic, PlotLabel → "Path of D - par set 1",
   AspectRatio → Automatic, PlotStyle → {{RGBColor[1, 0, 0]}}];
```



```
Show[plo1C, plo1D];
```



▪ actual position of C and D

```
pos1 = {xCpar, yCpar, xDpar, yDpar} /. φ → φ1 /. par1
```

{-18.7516, 80.4951, 43.0743, 84.1071}

```
pos1Error = val[[Range[1, 4]]] - pos1
```

{4.75158, 1.50494, 5.42566, -1.90712}


```
pos2 = {xCpar, yCpar, xDpar, yDpar} /. φ → φ2 /. par1
```

{-75.4833, 41.5667, -21.657, 72.1971}

```
pos2Error = val[[Range[5, 8]]] - pos2
```

{-0.516722, 0.633319, 0.657007, 0.202863}

```
pos3 = {xCpar, yCpar, xDpar, yDpar} /. φ → φ3 /. par1
```

{-55.3815, -21.8326, -34.4617, 36.4585}

```
pos3Error = val[[Range[9, 12]]] - pos3
```

{1.78151, -3.1674, 0.661728, -1.45853}

```
pos4 = {xCpar, yCpar, xDpar, yDpar} /. φ → φ4 /. par1
```

{-34.7382, 36.0955, 15.7901, 71.9059}

```
pos4Error = val[[Range[13, 16]]] - pos4
```

{0.738249, -1.09554, 1.70994, -0.405915}

▪ Plot of given C - D - points for the 4 states in x and y direction

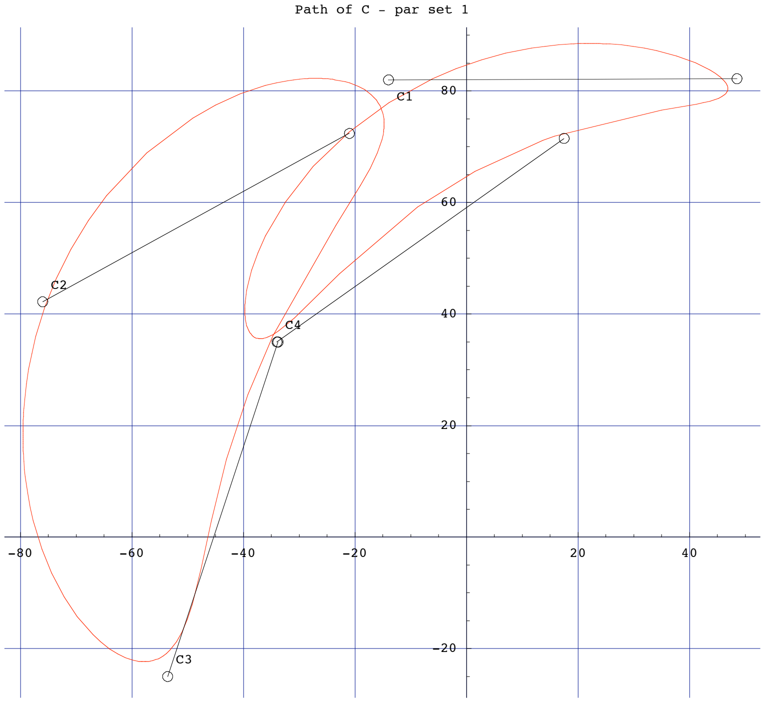
```
val = {-14, 82, 48.5, 82.2,
    -76, 42.2, -21, 72.4,
     -53.6, -25, -33.8, 35,
    -34, 35, 17.5, 71.5}
```

{-14, 82, 48.5, 82.2, -76, 42.2, -21, 72.4, -53.6, -25, -33.8, 35, -34, 35, 17.5, 71.5}

```
nfct = Length[val]
```

16

```
PloPoints = Graphics[{
    Circle[{val[[1]], val[[2]]}, lmax / 100],
    Circle[{val[[3]], val[[4]]}, lmax / 100],
    Text[C1, {val[[1]] + lmax / 30, val[[2]] - lmax / 30}],

    Circle[{val[[5]], val[[6]]}, lmax / 100],
    Circle[{val[[7]], val[[8]]}, lmax / 100],
    Text[C2, {val[[5]] + lmax / 30, val[[6]] + lmax / 30}],

    Circle[{val[[9]], val[[10]]}, lmax / 100],
    Circle[{val[[11]], val[[12]]}, lmax / 100],
    Text[C3, {val[[9]] + lmax / 30, val[[10]] + lmax / 30}],

    Circle[{val[[13]], val[[14]]}, lmax / 100],
    Circle[{val[[15]], val[[16]]}, lmax / 100],
    Text[C4, {val[[13]] + lmax / 30, val[[14]] + lmax / 30}],

    Line[{{val[[1]], val[[2]]}, {val[[3]], val[[4]]}}],
    Line[{{val[[5]], val[[6]]}, {val[[7]], val[[8]]}}],
    Line[{{val[[9]], val[[10]]}, {val[[11]], val[[12]]}}],
    Line[{{val[[13]], val[[14]]}, {val[[15]], val[[16]]}}]

   }];
Show[plo1C, plo1D, PloPoints];
```

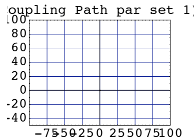


■ Animation of double-rocker mecha par 1 ($\psi \to f\psi$)

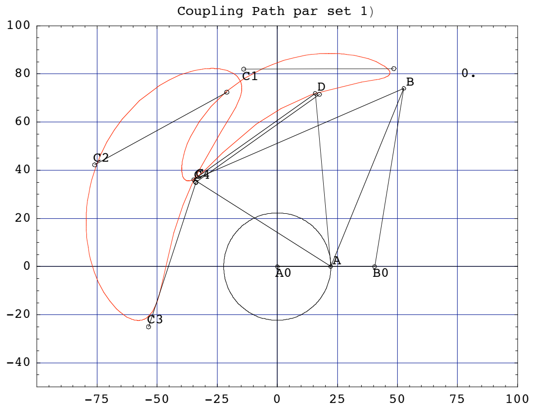
```
pa = par1
```

```
{A0x → 0, A0y → 0, γ0 → 0, a → 22.3, b → 75, c → 80, d → 40.5, kC → 67.5, κC → 80 °,
 kD → 72.2, κD → 0.479966, φ1 → 1.08, φ2 → 180 °, φ3 → 5.28319, φ4 → 360 °}
```

```
fψi = fψ;

rA0x = A0x /. pa;
rA0y = A0y /. pa;
rB0x = d Cos[γ0] /. pa;
rB0y = d Sin[γ0] /. pa;
rax = A0x + a Cos[φ + γ0] /. pa;
ray = A0y + a Sin[φ + γ0] /. pa;
rbx = A0x + b Cos[ψ + γ0] + d Cos[γ0] /. ψ → fψi /. pa;
rby = A0y + b Sin[ψ + γ0] + d Sin[γ0] /. ψ → fψi /. pa;
rcx = xC /. δ → fδ /. ψ → fψi /. pa;
rcy = yC /. δ → fδ /. ψ → fψi /. pa;
rdx = xD /. δ → fδ /. ψ → fψi /. pa;
rdy = yD /. δ → fδ /. ψ → fψi /. pa;
lmax = Max[a, b, c, d] /. pa;
Plo0 = Plot[0, {i, 0, 1},
   PlotRange → {{-100, 100}, {-50, 100}}, Frame → True, AspectRatio → Automatic,
   GridLines → Automatic, PlotLabel → "Coupling Path par set 1)"];
```




```
Do[
  PloMecha1 = Graphics[{
        Circle  [{rA0x,rA0y},a/.pa],

        Circle  [{rA0x,rA0y},lmax/100],
        Circle  [{rax,ray},lmax/100],
        Circle  [{rbx,rby},lmax/100],
        Circle  [{rcx,rcy},lmax/100],
        Circle  [{rdx,rdy},lmax/100],
        Circle  [{rB0x,rB0y},lmax/100],

        Line  [{{rA0x,rA0y}, {rB0x,rB0y}}],
        Line  [{{rA0x,rA0y}, {rax,ray}}],
        Line  [{{rB0x,rB0y}, {rbx,rby}}],
        Line  [{{rax,ray}, {rbx,rby}, {rcx,rcy}, {rax,ray}}],
        Line  [{{rcx,rcy}, {rdx,rdy}, {rax,ray}}],

        Text  [φ/Degree//N,{lmax, lmax}],
        Text  [A0,{rA0x+lmax/30, rA0y-lmax/30}],
        Text  [A ,{rax+lmax/30, ray+lmax/30}],
        Text  [B ,{rbx+lmax/30, rby+lmax/30}],
        Text  [C ,{rcx+lmax/30, rcy+lmax/30}],
        Text  [D ,{rdx+lmax/30, rdy+lmax/30}],
        Text  [B0,{rB0x+lmax/30,rB0y-lmax/30}]}];
   Show[Plo0, plo1C,plo1D, PloMecha1,PloPoints];
 ,{φ,0, 2 Pi, Pi/10}];
```



Part 2: Parameter-Optimization
ferror(p) -> min, ferror = sum((fct(p) - val)^2)

- **Given are point of B and K**

```
parReq = {A0x → 0, A0y → 0, γ0 → 0, d → 40.5}
```

```
{A0x → 0, A0y → 0, γ0 → 0, d → 40.5}
```

```
fct = {
    Simplify[xC /. δ -> fδ /. ψ -> fψ /. φ -> φ1 /. parReq],
    Simplify[yC /. δ -> fδ /. ψ -> fψ /. φ -> φ1 /. parReq],
    Simplify[xD /. δ -> fδ /. ψ -> fψ /. φ -> φ1 /. parReq],
    Simplify[yD /. δ -> fδ /. ψ -> fψ /. φ -> φ1 /. parReq],
    Simplify[xC /. δ -> fδ /. ψ -> fψ /. φ -> φ2 /. parReq],
    Simplify[yC /. δ -> fδ /. ψ -> fψ /. φ -> φ2 /. parReq],
    Simplify[xD /. δ -> fδ /. ψ -> fψ /. φ -> φ2 /. parReq],
    Simplify[yD /. δ -> fδ /. ψ -> fψ /. φ -> φ2 /. parReq],
    Simplify[xC /. δ -> fδ /. ψ -> fψ /. φ -> φ3 /. parReq],
    Simplify[yC /. δ -> fδ /. ψ -> fψ /. φ -> φ3 /. parReq],
    Simplify[xD /. δ -> fδ /. ψ -> fψ /. φ -> φ3 /. parReq],
    Simplify[yD /. δ -> fδ /. ψ -> fψ /. φ -> φ3 /. parReq],
    Simplify[xC /. δ -> fδ /. ψ -> fψ /. φ -> φ4 /. parReq],
    Simplify[yC /. δ -> fδ /. ψ -> fψ /. φ -> φ4 /. parReq],
    Simplify[xD /. δ -> fδ /. ψ -> fψ /. φ -> φ4 /. parReq],
    Simplify[yD /. δ -> fδ /. ψ -> fψ /. φ -> φ4 /. parReq]

  };
```

```
nfct = Length[fct]
```

```
16
```

```
fct[[4]] //. par1 // N
```

```
84.1071
```


- **setup the error fct.**

```
ferror = Sum[(fct[[i]] - val[[i]])^2, {i, 1, nfct}] // N;
```

```
ferror /. par1 // N
```

```
79.6634
```

```
par1
```

```
{A0x → 0, A0y → 0, γ0 → 0, a → 22.3, b → 75, c → 80, d → 40.5, kC → 67.5, κC → 80 °,
 kD → 72.2, κD → 0.479966, φ1 → 1.08, φ2 → 180 °, φ3 → 5.28319, φ4 → 360 °}
```

- **Find the solution of p**

```
pinit = {{φ1, 1.08}, {φ2, Pi}, {φ3, 5.28}, {φ4, 360 Degree}, {a, 22.3},
   {b, 75}, {c, 80}, {kC, 67.5}, {kD, 72.2}, {κC, 80 Degree}, {κD, 27.5 Degree}}
```

```
{{φ1, 1.08}, {φ2, π}, {φ3, 5.28}, {φ4, 360 °}, {a, 22.3},
 {b, 75}, {c, 80}, {kC, 67.5}, {kD, 72.2}, {κC, 80 °}, {κD, 0.479966}}
```

```
parOpt = FindMinimum[ferror, pinit]
```

```
{0.267362, {φ1 → 1.02097, φ2 → 3.12532, φ3 → 5.31852, φ4 → 6.27636, a → 23.6085,
  b → 86.7703, c → 90.5271, kC → 67.2628, kD → 71.9695, κC → 1.32813, κD → 0.393111}}
```

- **Tests**

```
φ2 / Degree /. parOpt[[2, 2]]
```

```
179.068
```

```
xC /. δ -> fδ /. ψ -> fψ /. φ -> φ2 /. parReq /. parOpt[[2]]
```

```
-76.2139
```

```
yC /. δ -> fδ /. ψ -> fψ /. φ -> φ2 /. parReq /. parOpt[[2]]
```

```
42.2963
```

- **Test of functions by a par set 2: optimized solution - non-scaled**

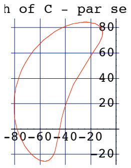
```
par2 = Join[parReq, parOpt[[2]]]
```

```
{A0x → 0, A0y → 0, γ0 → 0, d → 40.5, φ1 → 1.02097,
 φ2 → 3.12532, φ3 → 5.31852, φ4 → 6.27636, a → 23.6085, b → 86.7703,
 c → 90.5271, kC → 67.2628, kD → 71.9695, κC → 1.32813, κD → 0.393111}
```

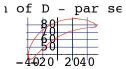
```
par1 = par2;  (*  we use par1 again! *)
```

- **plot of mechanism**

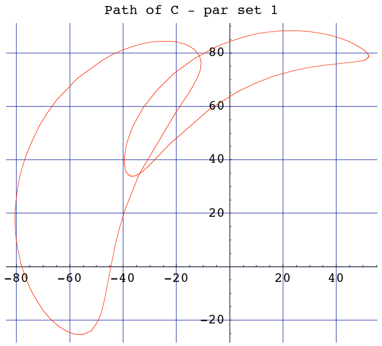
```
xCpar = xC /. δ -> fδ /. ψ -> fψ /. par1;
yCpar = yC /. δ -> fδ /. ψ -> fψ /. par1;
plo1C = ParametricPlot[{xCpar, yCpar}, {φ, 0, 2 Pi},
   GridLines → Automatic, PlotLabel → "Path of C - par set 1",
   AspectRatio → Automatic, PlotStyle → {{RGBColor[1, 0, 0]}}];
```




```
xDpar = xD /. δ -> fδ /. ψ -> fψ /. par1;
yDpar = yD /. δ -> fδ /. ψ -> fψ /. par1;
plo1D = ParametricPlot[{xDpar, yDpar}, {φ, 0, 2 Pi},
   GridLines → Automatic, PlotLabel → "Path of D - par set 1",
   AspectRatio → Automatic, PlotStyle → {{RGBColor[1, 0, 0]}}];
```




```
Show[plo1C, plo1D];
```



- **actual position of C and D**

```
pos1 = {xCpar, yCpar, xDpar, yDpar} /. φ → φ1 /. par1
```

{-14.2008, 81.9357, 48.686, 82.2443}

```
pos1Error = val[[Range[1, 4]]] - pos1
```

{0.20082, 0.0643042, -0.185992, -0.0442829}


```
pos2 = {xCpar, yCpar, xDpar, yDpar} /. φ → φ2 /. par1
```

{-76.2139, 42.2963, -20.9477, 72.3045}

```
pos2Error = val[[Range[5, 8]]] - pos2
```

{0.213865, -0.0962714, -0.0523158, 0.0954686}

```
pos3 = {xCpar, yCpar, xDpar, yDpar} /. φ → φ3 /. par1
```

{-53.5941, -24.8293, -33.8186, 34.8681}

```
pos3Error = val[[Range[9, 12]]] - pos3
```

{-0.00591554, -0.170671, 0.018636, 0.131941}

```
pos4 = {xCpar, yCpar, xDpar, yDpar} /. φ → φ4 /. par1
```

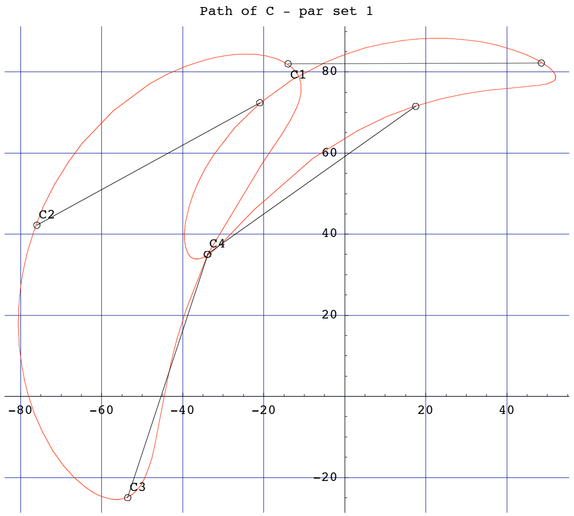
{-33.7545, 34.9648, 17.4029, 71.5403}

```
pos4Error = val[[Range[13, 16]]] - pos4
```

{-0.245511, 0.0351586, 0.097096, -0.0403465}

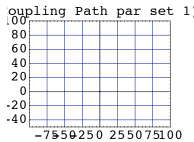
■ Plot of given C - D - points for the 4 states in x and y direction

```
PloPoints = Graphics[{
    Circle[{val[[1]], val[[2]]}, lmax / 100],
    Circle[{val[[3]], val[[4]]}, lmax / 100],
    Text[C1, {val[[1]] + lmax / 30, val[[2]] - lmax / 30}],

    Circle[{val[[5]], val[[6]]}, lmax / 100],
    Circle[{val[[7]], val[[8]]}, lmax / 100],
    Text[C2, {val[[5]] + lmax / 30, val[[6]] + lmax / 30}],

    Circle[{val[[9]], val[[10]]}, lmax / 100],
    Circle[{val[[11]], val[[12]]}, lmax / 100],
    Text[C3, {val[[9]] + lmax / 30, val[[10]] + lmax / 30}],

    Circle[{val[[13]], val[[14]]}, lmax / 100],
    Circle[{val[[15]], val[[16]]}, lmax / 100],
    Text[C4, {val[[13]] + lmax / 30, val[[14]] + lmax / 30}],

    Line[{{val[[1]], val[[2]]}, {val[[3]], val[[4]]}}],
    Line[{{val[[5]], val[[6]]}, {val[[7]], val[[8]]}}],
    Line[{{val[[9]], val[[10]]}, {val[[11]], val[[12]]}}],
    Line[{{val[[13]], val[[14]]}, {val[[15]], val[[16]]}}]

   }, PlotStyle → {{RGBColor[1, 0, 1]}}];
Show[plo1C, plo1D, PloPoints];
```

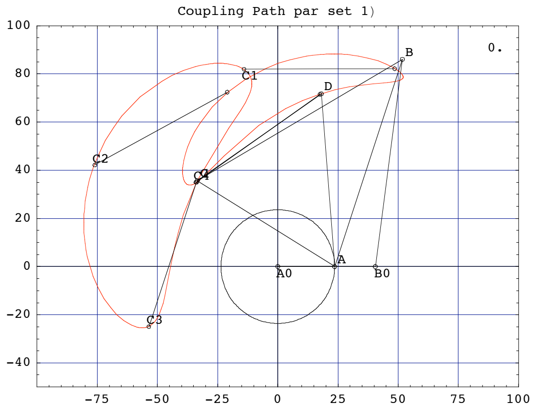


■ Animation of double-rocker mecha par 1 (ψ→fψ)

```
pa = par1

{A0x → 0, A0y → 0, γ0 → 0, d → 40.5, φ1 → 1.02097,
 φ2 → 3.12532, φ3 → 5.31852, φ4 → 6.27636, a → 23.6085, b → 86.7703,
 c → 90.5271, kC → 67.2628, kD → 71.9695, κC → 1.32813, κD → 0.393111}

fψi = fψ;

rA0x = A0x /. pa;
rA0y = A0y /. pa;
rB0x = d Cos[γ0] /. pa;
rB0y = d Sin[γ0] /. pa;
rax = A0x + a Cos[φ + γ0] /. pa;
ray = A0y + a Sin[φ + γ0] /. pa;
rbx = A0x + b Cos[ψ + γ0] + d Cos[γ0] /. ψ → fψi /. pa;
rby = A0y + b Sin[ψ + γ0] + d Sin[γ0] /. ψ → fψi /. pa;
rcx = xC /. δ → fδ /. ψ → fψi /. pa;
rcy = yC /. δ → fδ /. ψ → fψi /. pa;
rdx = xD /. δ → fδ /. ψ → fψi /. pa;
rdy = yD /. δ → fδ /. ψ → fψi /. pa;
lmax = Max[a, b, c, d] /. pa;
Plo0 = Plot[0, {i, 0, 1},
   PlotRange → {{-100, 100}, {-50, 100}}, Frame → True, AspectRatio → Automatic,
   GridLines → Automatic, PlotLabel → "Coupling Path par set 1)"];
```




```
Do[
  PloMecha1 = Graphics[{
        Circle  [{rA0x,rA0y},a/.pa],

        Circle  [{rA0x,rA0y},lmax/100],
        Circle  [{rax,ray},lmax/100],
        Circle  [{rbx,rby},lmax/100],
        Circle  [{rcx,rcy},lmax/100],
        Circle  [{rdx,rdy},lmax/100],
        Circle  [{rB0x,rB0y},lmax/100],

        Line  [{{rA0x,rA0y}, {rB0x,rB0y}}],
        Line  [{{rA0x,rA0y}, {rax,ray}}],
        Line  [{{rB0x,rB0y}, {rbx,rby}}],
        Line  [{{rax,ray}, {rbx,rby}, {rcx,rcy}, {rax,ray}}],
        Line  [{{rcx,rcy}, {rdx,rdy}, {rax,ray}}],

        Text  [φ/Degree//N,{lmax, lmax}],
        Text  [A0,{rA0x+lmax/30, rA0y-lmax/30}],
        Text  [A ,{rax+lmax/30, ray+lmax/30}],
        Text  [B ,{rbx+lmax/30, rby+lmax/30}],
        Text  [C ,{rcx+lmax/30, rcy+lmax/30}],
        Text  [D ,{rdx+lmax/30, rdy+lmax/30}],
        Text  [B0,{rB0x+lmax/30,rB0y-lmax/30}]}];
   Show[Plo0, plo1C,plo1D, PloMecha1,PloPoints];
 ,{φ,0, 2 Pi, Pi/10}];
```



Further Evaluations of the mechanism

```
par2
```

{A0x → 0, A0y → 0, γ0 → 0, d → 40.5, φ1 → 1.02097,
φ2 → 3.12532, φ3 → 5.31852, φ4 → 6.27636, a → 23.6085, b → 86.7703,
c → 90.5271, kC → 67.2628, kD → 71.9695, κC → 1.32813, κD → 0.393111}

```
φ1 / Degree /. par2
```

58.4975

```
φ2 / Degree /. par2
```

179.068

```
φ3 / Degree /. par2
```

304.728

```
φ4 / Degree /. par2
```

359.609

■ Toggle angle

```
φi = Pi + ArcCos[(d^2 + (c - a)^2 - b^2) / (2 (c - a) d)];
φa = ArcCos[(d^2 + (c + a)^2 - b^2) / (2 (c + a) d)];
fφ0 = φi - φa
```

$$\pi + \text{ArcCos}\left[\frac{-b^2 + (-a+c)^2 + d^2}{2\,(-a+c)\,d}\right] - \text{ArcCos}\left[\frac{-b^2 + (a+c)^2 + d^2}{2\,(a+c)\,d}\right]$$


```
ψi = Pi - ArcCos[(d^2 + b^2 - (c - a)^2) / (2 b d)];
ψa = Pi - ArcCos[(d^2 + b^2 - (c + a)^2) / (2 b d)];
fψ0 = ψi - ψa
```

$$-\text{ArcCos}\left[\frac{b^2 - (-a+c)^2 + d^2}{2\,b\,d}\right] + \text{ArcCos}\left[\frac{b^2 - (a+c)^2 + d^2}{2\,b\,d}\right]$$

```
fφ0 / Degree /. par2
```

245.629

```
fψ0 / Degree /. par2
```

75.1608

■ Transmission angle

```
μmin1 = ArcCos[(c^2 + b^2 - (d - a)^2) / (2 b c)]
```

$$\text{ArcCos}\left[\frac{b^2 + c^2 - (-a+d)^2}{2\,b\,c}\right]$$

```
μmin2 = Pi - ArcCos[(c^2 + b^2 - (d + a)^2) / (2 b c)]
```

$$\pi - \text{ArcCos}\left[\frac{b^2 + c^2 - (a+d)^2}{2\,b\,c}\right]$$

```
μmin1 / Degree /. par2
```

10.6617

```
180 - μmin2 / Degree /. par2
```

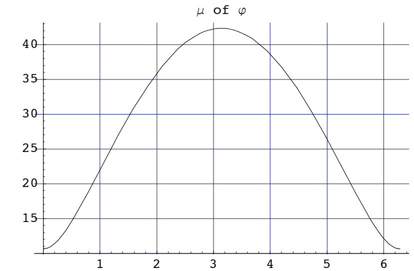
42.329

Plot of $\mu(\varphi)$

```
μ = ArcCos[(b^2 + c^2 - Δ^2) / (2 b c)]
```

$$\text{ArcCos}\left[\frac{-a^2 + b^2 + c^2 - d^2 + 2\,a\,d\,\text{Cos}[\varphi]}{2\,b\,c}\right]$$

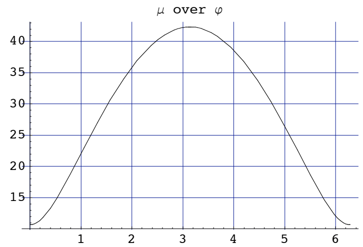
```
Plot[μ / Degree /. par2, {φ, 0, 2 Pi}, GridLines -> Automatic, PlotLabel -> "μ of φ"];
```



```
μfct[φ_, a_, b_, c_, d_] := If[ArcCos[(-a^2 + b^2 + c^2 - d^2 + 2 a d Cos[φ])/(2 b c)] < 90 Degree,
   ArcCos[(-a^2 + b^2 + c^2 - d^2 + 2 a d Cos[φ])/(2 b c)], Pi - ArcCos[(-a^2 + b^2 + c^2 - d^2 + 2 a d Cos[φ])/(2 b c)]];
μfct[0, a /. par2, b /. par2, c /. par2, d /. par2] / Degree
```

```
10.6617
```

```
Plot[μfct[φ, a /. par2, b /. par2, c /. par2, d /. par2] / Degree,
  {φ, 0, 2 Pi}, GridLines -> Automatic, PlotLabel -> "μ over φ"];
```



■ **Linear Velocity of C and D - scaling: Ms = 4.05/43**

```
ω = 2 Pi n / 60 /. n → 100 // N
```

```
10.472
```

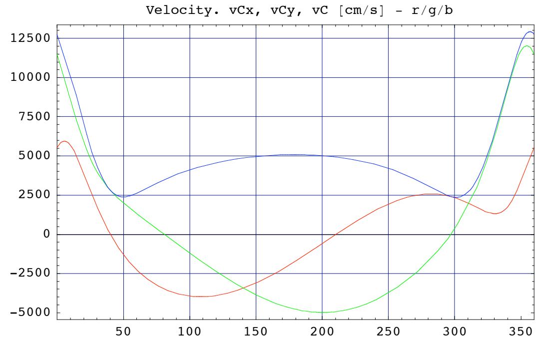
```
Ms = 4.05 / 43
```

```
0.094186
```

```
vCx = D[Simplify[xCpar / Ms], φ] ω;
vCy = D[Simplify[yCpar / Ms], φ] ω;
vC = Sqrt[vCx ^ 2 + vCy ^ 2];
```



```
φ = phiG * Degree;
Plot[{vCx, vCy, vC}, {phiG, 0, 360}, Frame → True,
  GridLines → Automatic, PlotLabel → " Velocity. vCx, vCy, vC [cm/s] - r/g/b",
  PlotRange → {{0, 360}, Automatic},
  PlotStyle → {{RGBColor[1, 0, 0]}, {RGBColor[0, 1, 0]}, {RGBColor[0, 0, 1]}}];
Clear[
  φ];
```

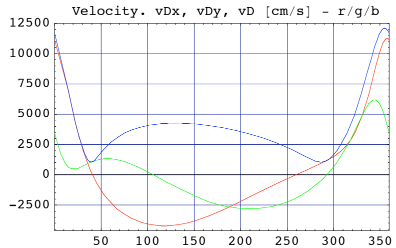



```
vC /. φ → 0
```

```
12753.9
```

```
vDx = D[Simplify[xDpar / Ms], φ] ω;
vDy = D[Simplify[yDpar / Ms], φ] ω;
vD = Sqrt[vDx ^ 2 + vDy ^ 2];
```

```
φ = phiG * Degree;
Plot[{vDx, vDy, vD}, {phiG, 0, 360}, Frame → True,
  GridLines → Automatic, PlotLabel → " Velocity. vDx, vDy, vD [cm/s] - r/g/b",
  PlotRange → {{0, 360}, Automatic},
  PlotStyle → {{RGBColor[1, 0, 0]}, {RGBColor[0, 1, 0]}, {RGBColor[0, 0, 1]}}];
Clear[
  φ];
```




```
vD /. φ → 355 Degree
```

```
12101.2
```

```
vD /. φ → φa /. par2
```

```
1060.75
```

Note: for φi or φa the velocity of C and D are not zero!

▪ Find the torque M2 at crank due to gravity load at the box CM with Fg = m g

```
Fg = m g /. {m → 60, g → 9.81}
```

588.6

▪ find vCMx,y:

```
κCD = κC - κD
```

$\kappa C - \kappa D$

```
CD = Sqrt[-(2 kC kD Cos[κCD]) + (kC^2 + kD^2)]
```

$\sqrt{kC^2 + kD^2 - 2\, kC\, kD\, \mathrm{Cos}[\kappa C - \kappa D]}$

```
CD /. par2
```

62.8876

```
lCM = Sqrt[(CD/2)^2 + (DE/2)^2]
```

$\sqrt{\frac{DE^2}{4} + \frac{1}{4}\left(kC^2 + kD^2 - 2\, kC\, kD\, \mathrm{Cos}[\kappa C - \kappa D]\right)}$

```
αCM = ArcCos[CD/2/lCM]
```

$\mathrm{ArcCos}\left[\frac{\sqrt{kC^2 + kD^2 - 2\, kC\, kD\, \mathrm{Cos}[\kappa C - \kappa D]}}{2\sqrt{\frac{DE^2}{4} + \frac{1}{4}\left(kC^2 + kD^2 - 2\, kC\, kD\, \mathrm{Cos}[\kappa C - \kappa D]\right)}}\right]$

```
αCM / Degree /. DE → 42 /. par2
```

33.7373

```
ηC = ArcCos[(kD^2 + CD^2 - kC^2) / (2 kD CD)]
```

$\mathrm{ArcCos}\left[\frac{2\, kD^2 - 2\, kC\, kD\, \mathrm{Cos}[\kappa C - \kappa D]}{2\, kD \sqrt{kC^2 + kD^2 - 2\, kC\, kD\, \mathrm{Cos}[\kappa C - \kappa D]}}\right]$

```
ηC / Degree /. DE → 42 /. par2
```

59.3828

```
ηCM = ηC + αCM /. DE → 42 /. par2
```

1.62525

```
ηCM / Degree
```

93.1201

```
kCM = Sqrt[-(2 kD lCM Cos[ηCM]) + (kD^2 + lCM^2)] /. DE → 42 /. par2
```

83.0997


```
κDCM = ArcCos[(kD^2 + kCM^2 - lCM^2) / (2 kD kCM)] /. DE → 42 /. par2
```

0.471631

```
κDCM / Degree
```

27.0225

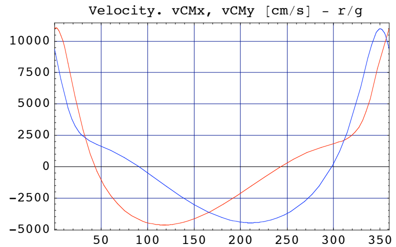
```
κCM = κD + κDCM /. par2
```

0.864742

```
κCM / Degree
```

49.5461

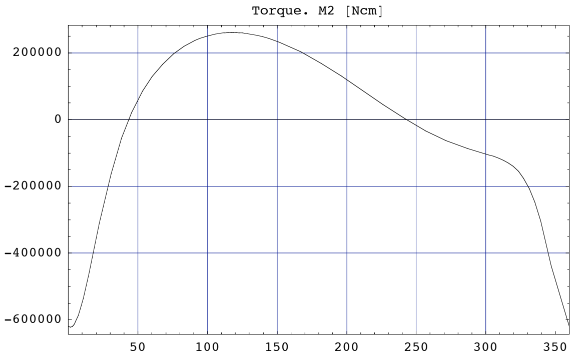
```
xCMpar = A0x + a Cos[φ + γ0] + kCM Cos[κCM + δ + γ0] /. δ -> fδ /. ψ -> fψ /. par2;
yCMpar = A0y + a Sin[φ + γ0] + kCM Sin[κCM + δ + γ0] /. δ -> fδ /. ψ -> fψ /. par2;

vCMx = D[Simplify[xCMpar / Ms], φ] ω;
vCMy = D[Simplify[yCMpar / Ms], φ] ω;

φ = phiG * Degree;
Plot[{vCMx, vCMy}, {phiG, 0, 360}, Frame → True, GridLines → Automatic,
  PlotLabel → " Velocity. vCMx, vCMy [cm/s] - r/g", PlotRange → {{0, 360}, Automatic},
  PlotStyle → {{RGBColor[1, 0, 0]}, {RGBColor[0, 0, 1]}}];
Clear[
  φ];
```



▪ Power of Fg: Fg vCMx + M2 ω = 0

```
M2 = -Fg vCMx / ω;
```

```
φ = phiG * Degree;
Plot[{M2}, {phiG, 0, 360}, Frame → True, GridLines → Automatic,
  PlotLabel → " Torque. M2 [Ncm] ", PlotRange → {{0, 360}, Automatic}];
Clear[
  φ];
```




```
M2 /. φ → 2 Degree
```

```
-621398.
```

```
M2 /. φ → 120 Degree
```

```
261506.
```