ModSim Vorlesung MFB420, Prof. Wallrapp, HM

■ Übung 4/10

■ Nichtlineare Rechnung

```
hdot = 1/ρ/A(mzdot - ρ*AL*Sqrt[2*g*h[t]])
```

$$\frac{\text{mzdot} - \sqrt{2}\ \text{AL}\ \rho \sqrt{g\,h[t]}}{A\,\rho}$$

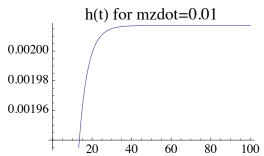
```
par = {A → 0.12^2 Pi / 4, AL → 0.008^2 Pi / 4, g → 9.81, hmax → 0.2, ρ → 1000}
```

{A → 0.0113097, AL → 0.0000502655, g → 9.81, hmax → 0.2, ρ → 1000}

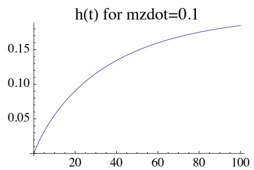
```
hdotpar = hdot /. par
```

$0.0884194 \left(\text{mzdot} - 0.222648 \sqrt{h[t]}\right)$

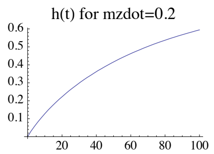
```
erg01 = NDSolve[{h'[t] == hdotpar /. mzdot → 0.01, h[0] == 0}, h, {t, 0, 100}];
herg01 = h[t] /. erg01[[1]];
plo01 = Plot[herg01, {t, 0, 100}, PlotLabel → "h(t) for mzdot=0.01"]
```



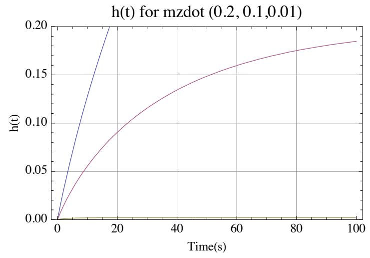

```
erg1 = NDSolve[{h'[t] == hdotpar /. mzdot → 0.1, h[0] == 0}, h, {t, 0, 100}];
herg1 = h[t] /. erg1[[1]];
plo1 = Plot[herg1, {t, 0, 100}, PlotLabel → "h(t) for mzdot=0.1"]
```




```
erg2 = NDSolve[{h'[t] == hdotpar /. mzdot → 0.2, h[0] == 0}, h, {t, 0, 100}];
herg2 = h[t] /. erg2[[1]];
plo2 = Plot[herg2, {t, 0, 100}, PlotLabel → "h(t) for mzdot=0.2"]
```




```
Plot[{herg2, herg1, herg01}, {t, 0, 100}, Frame → True,
 FrameLabel → {"Time(s)", "h(t)"}, PlotLabel → "h(t) for mzdot (0.2, 0.1,0.01)",
 GridLines → Automatic, PlotRange → {Automatic, {0, 0.2}}]
```



Achtung: bei mzdot=0.2 laeuft der Tank ueber!

■ Lineare Rechnung mLdot => k h: für h* = 0.1, k = Steigung

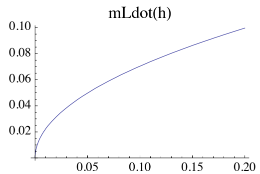
```
mLdot = AL ρ Sqrt[2 g] Sqrt[h[t]]
```

$\sqrt{2}\ \text{AL}\ \sqrt{g}\ \rho \sqrt{h[t]}$

```
mLdotpar = mLdot /. par
```

$0.222648 \sqrt{h[t]}$

```
plomLdot = Plot[mLdotpar, {h[t], 0, 0.2}, PlotLabel → "mLdot(h)"]
```




```
dmLdot = Simplify[D[mLdot, h[t]]]
```

$$\frac{\text{AL}\sqrt{g}\ \rho}{\sqrt{2}\ \sqrt{h[t]}}$$

```
k = dmLdot /. par /. h[t] → 0.1
```

0.352038

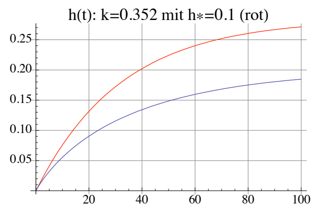
Die Steigung beträgt k = 0.352038

```
dgl = (h'[t] == mzdot / A / ρ - k / A / ρ * h[t]) /. par /. mzdot → 0.1
```

$h'[t] == 0.00884194 - 0.031127\, h[t]$

```
erg3lin = NDSolve[{dgl, h[0] == 0}, h, {t, 0, 100}];
herg3lin = h[t] /. erg3lin[[1]];
plo3lin = Plot[herg3lin, {t, 0, 100},
   PlotStyle → {{RGBColor[1, 0, 0]}}, PlotLabel → "h(t): k=0.352 mit h*=0.1 (rot)"];
```

```
Show[plo3lin, plo1, GridLines → Automatic]
```



Achtung: diese Linearisierung (k = 0.352) liefert grosse Abweichungen bei h(t).

Lineare Rechnung mit minimalem Fehler

```
mLdotpar
```

$0.222648\sqrt{h[t]}$

```
datay = Table[mLdotpar, {h[t], 0, 0.2, 0.01}]
```

```
{0., 0.0222648, 0.0314872, 0.0385638, 0.0445297, 0.0497857, 0.0545375,
 0.0589072, 0.0629744, 0.0667945, 0.0704076, 0.0738441, 0.0771276, 0.080277,
 0.0833074, 0.0862313, 0.0890593, 0.0918002, 0.0944617, 0.0970501, 0.0995713}
```

```
datax = Table[0.01 * i, {i, 0, 20}]
```

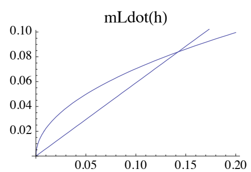
```
{0, 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09,
 0.1, 0.11, 0.12, 0.13, 0.14, 0.15, 0.16, 0.17, 0.18, 0.19, 0.2}
```

```
data = Transpose[{datax, datay}];
k = Fit[data, {0, h[t]}, h[t]] / h[t]
```

```
0.590209
```

```
plomLdotlin = Plot[k h[t], {h[t], 0, 0.2`}, PlotLabel → "k(h)"];
```

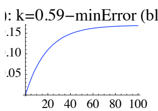
```
Show[plomLdot, plomLdotlin]
```



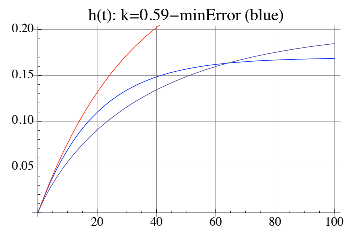

```
dgl = (h'[t] == mzdot / A / ρ - k / A / ρ * h[t]) /. par /. mzdot → 0.1
```

$h'[t] == 0.00884194 - 0.052186\, h[t]$

```
erg4lin = NDSolve[{dgl, h[0] == 0}, h, {t, 0, 100}];
herg4lin = h[t] /. erg4lin[[1]];
plo4lin = Plot[herg4lin, {t, 0, 100},
  PlotStyle → {{RGBColor[0, 0, 1]}}, PlotLabel → "h(t): k=0.59-minError (blue)"]
```




```
Show[plo4lin, plo3lin, plo1, GridLines → Automatic, PlotRange → {Automatic, {0, 0.2}}]
```



Mit dieser linearen Funktion k = 0.59 aus minError erhält man eine Loesung, die für die gesamte Hoehe brauchbar ist!